British Astronomy and Astrophysics Olympiad 2024-2025

Astro Round 1 Section 2

Tuesday 28th January 2025

This question paper must not be photographed or taken out of the exam room

Instructions

Time: 1 hour.

Questions: Answer any **TWO** questions, worth 20 marks each to give a total of 40 marks for this section (there are FIVE questions available).

Solutions: Answers and calculations are to be written on loose paper. START EACH QUESTION ON A NEW PAGE. Students should ensure their **name** and **school** is clearly written on the **first** answer sheet and that **all** pages are numbered. A standard formula booklet with standard physical constants may be used if desired.

Clarity: Solutions must be written legibly, in black or blue pen, and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Sitting the paper: There are two options for sitting the Astro Round 1:

1. Section 1 and Section 2 may be sat in one session of 2 hours. Section 1 should be collected in after 1 hour and only then should Section 2 be given out.
2. Section 1 and Section 2 are sat in entirely separate sessions of 1 hour each.

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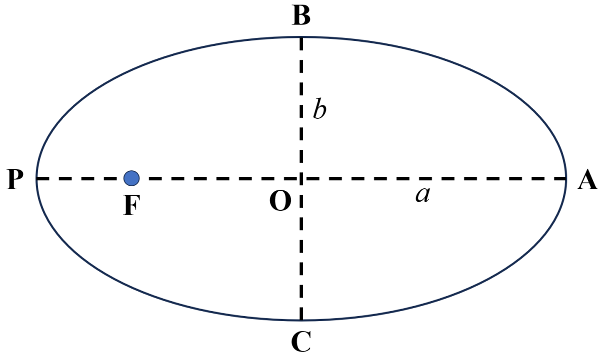
Important Constants

Constant	**Symbol**	**Value**
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Sun	$R_{\odot}$	$6.96 \times 10^{8}\,\mathrm{m}$
Radius of the Earth	$R_{\oplus}$	$6.37 \times 10^{6}\,\mathrm{m}$
Mass of the Sun	$M_{\odot}$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$M_{\oplus}$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_{\odot}$	$3.83 \times 10^{26}\,\mathrm{W}$
Absolute magnitude of the Sun	$\mathcal{M}_{\odot}$	4.74
Hubble constant	H_0	70 km s^{-1} Mpc^{-1}
Stephan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k_{B}	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck's constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Proton rest mass	m_{p}	1.67×10^{-27} kg
Electron rest mass	m_{e}	9.11×10^{-31} kg
Wien's displacement law	$\lambda_{\mathrm{max}}T$	2.90×10^{-3} m K
Avagadro's constant	N_A	6.02×10^{23} mol^{-1}

Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from **F**)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from **F**)
πab	area of the ellipse

Kepler's Third Law:

$$T^2 = \frac{4\pi^2}{GM} a^3$$

Vis-Viva Equation:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Wien's Displacement Law:

$$\lambda_{\max} T = \text{constant}$$

Stephan-Boltzmann Law:

$$L = 4\pi R^2 \sigma T^4$$

Brightness (Intensity):

$$b = \frac{L}{4\pi d^2}$$

Magnitudes:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

$$m - \mathcal{M} = 5\log\left(\frac{d}{10}\right)$$

Distance-Parallax Relation:

$$d = \frac{1}{p}$$

Rayleigh Criterion:

$$\theta = \frac{1.22\lambda}{D}$$

Redshift:

$$z = \frac{\Delta\lambda}{\lambda_{\text{emit}}} \approx \frac{v}{c}$$

Hubble's Law:

$$v = H_0 d$$

Qu 2. Spaceship Strategies

In this paper there are FIVE questions and you should attempt only TWO.

Consider a spaceship in orbit around a star with mass M. The orbit has a semi-major axis of a and an eccentricity of e.

a. Derive an expression for:

   (i) The speed of the spaceship at periapsis, v_p.

   (ii) The speed of the spaceship at apoapsis, v_a.

   (iii) The ratio $\frac{v_p}{v_a}$ expressed in its simplest form.

[3]

Three spaceships, A, B and C, are orbiting the same star with mass M. Each has an orbit with the same semi-major axis a_1 and eccentricity e, although their orbital planes are rotated so that they do not hit each other. They all receive a call from Galactic Command to leave this star system.

b. The captain of Spaceship A fires their rockets once whilst at periapsis to reach (effectively instantaneously) escape velocity. Derive an expression for the increase in speed of the spaceship in the form $\Delta v_A = k_A \sqrt{\frac{GM}{a_1}}$ where k_A is a function of e to be found.

[3]

c. The captain of Spaceship B does a very similar thing to the captain of Spaceship B, however they fire their rockets once whilst at apoapsis. State the increase in the speed of the spaceship in the form $\Delta v_B = k_B \sqrt{\frac{GM}{a_1}}$ where k_B is a new function of e to be found.

[1]

d. The captain of Spaceship C is in no hurry to leave the system and is keen to minimise their fuel use in each burst of the rockets. Their science officer suggests that they can do better than Spaceship B by also firing their rockets whilst at apoapsis to increase their speed, but only enough to make this point in the orbit now the periapsis of a new orbit with a larger semi-major axis, a_2, but the same eccentricity.

   (i) Derive an expression for the increase in the speed of the spaceship in the form $\Delta v_1 = k_1 \sqrt{\frac{GM}{a_1}}$ where k_1 is a function of e to be found, and confirm that $k_1 \leq k_B$ for all e.

   (ii) Whilst the science officer is celebrating, a junior officer points out that Spaceship B is escaping the system whilst they are still in orbit, and that a fairer comparison is to consider all the fuel used across all the steps until they escape too. The captain therefore repeats the process in each new orbit, applying the strategy every time they get to a new apoapsis, until eventually (after a very long time!) they are too no longer bound to the system. Derive an expression for the total change in speed their spaceship's rockets had to produce throughout this strategy, again keeping the same form of the final answer such that $\Delta v_c = \sum_{i=1}^{\infty} \Delta v_i = k_c \sqrt{\frac{GM}{a_1}}$ where k_c is a function of e to be found. Was this strategy more fuel efficient than that of Spaceship B for any values of e?

[6]

In the star system is the planet Stevenson, the inhabitants of which are keen astronomers, and which orbits the star in a circular orbit. In their language, the distance from the star to the planet Stevenson is defined as 1 Char (their equivalent of the au) and the time it takes them to complete one orbit of their star is 1 Lotte (their equivalent of the year).

e. The astronomers on Stevenson see the rocket of Spaceship C fire its first burst of its rocket whilst the spaceship is at opposition (i.e. the star, Stevenson and the spaceship all form a perfectly straight line, in that order) with a luminosity of L. By cosmic coincidence, its happens to be that the values of a_1 (in Char) and e are such that the astronomers see **every** burst of the rockets by the spaceship and the spaceship is **always** at opposition when they do so.

   (i) Given that $a_1 < 5$ Char and $e < 0.9$, find the only values of a_1 and e that allow this outcome.

   (ii) What is the time elapsed (in Lottes) between the first and second bursts?

   (iii) What is the change in the apparent magnitude of the rocket burst between the first and second bursts?

[7]

Qu 3. Black Dwarfs

Zhang et al. (2024) used gravitational microlensing to discover an Earth-like planet orbiting a white dwarf in a system 1.33 kpc away. The white dwarf has a mass of $0.49\ M_\odot$ and the planet has a mass of $1.87\ M_\oplus$ and an orbital radius of 2.07 au - this is very similar to what some scientists believe might be the final configuration of our own Solar System and so provides a glimpse into our long-term future.

The apparent magnitude of the white dwarf was measured to be 24.0 and careful computer modelling for a white dwarf of that mass shows that $\log_{10}(g/\mathrm{m\,s^{-2}}) = 5.771$.

a. Determine the radius of the white dwarf in units of Earth radii, $R_\oplus$.

[2]

b. Calculate the effective surface temperature, T_{eff}, and suggest what colour it might appear to a human astronaut passing by the system.

[4]

c. If the planet has an Earth-like albedo of $\alpha = 0.3$ (meaning the surface reflects 30% of the incident radiation), what would be the expected equilibirum temperature?

[3]

White dwarfs are born very hot, however radiate their energy away through their luminosity. How their T_{eff} varies with the age of the white dwarf, t_{WD}, is a highly complex process due to the non-trivial changes to the internal structure of the white dwarf as it cools, but a very simplified model for a $\sim 0.5 M_\odot$ white dwarf assumes it is a simple power law and follows the equation:

$$\log_{10}(T_{eff}/\mathrm{K}) = -0.3153 \log_{10}(t_{WD}/\mathrm{yr}) + 6.7322\ .$$

d. Use your answer to part b. to show that the age of the white dwarf is about 2 Gyr. [1 Gyr $= 10^9$ yr]

[2]

Often at GCSE and A Level in discussions of stellar evolution, the sequence for a Sun-like star finishes with a white dwarf that cools down to a black dwarf. There is no official definition of how cool a white dwarf needs to become before it is considered a "black" dwarf, but an upper limit to that temperature can be given by the Draper point, which is the temperature at which the human eye would struggle to distinguish any visible light being emitted from an object. This corresponds to a peak black body wavelength of 3.64 µm.

e. Estimate the minimum time for a $0.5\ M_\odot$ white dwarf to cool down to a 'black' dwarf, giving your answer in units of the age of the Universe, $t_0 = 13.8$ Gyr.

[2]

The very long term fate of this planet can go one of two ways: gravitational radiation drives orbital decay into the white (black) dwarf on a time scale given by

$$\tau_{GR} = \frac{2\pi a}{v}\left(\frac{v}{c}\right)^{-5}$$

where a is the orbital radius, v is the orbital speed, and c is the speed of light, or alternatively planets can be ejected from their orbits by a close interaction with a passing star with a time scale of

$$\tau_{eject} = (n\sigma v_{rel})^{-1}$$

where n is the number density of stars (about ~ 0.1 pc^{-3}), σ is the area of the orbit ($\approx \pi a^2$) and v_{rel} is the relative velocity between the planet and the star (assumed ~ 100 km s^{-1}).

f. Which outcome is the most likely for this planet? What orbital radius gives equal time scales?

[7]

Qu 4. The International Liquid Mirror Telescope

The International Liquid Mirror Telescope (ILMT) is a research telescope located in Uttarakhand, India (latitude $29.4°$). It was completed in 2022 as the result of a collaboration between researchers from around the world. As the name suggests, instead of the solid primary mirror seen in most reflecting telescopes, the ILMT gathers light using a rotating pool of mercury, visible in Figure 1. This is possible because the surface of a column of fluid revolving at uniform angular velocity forms a paraboloid, which is exactly the shape used for the primary mirror of a reflecting telescope.

Figure 1: The image of the completed liquid mirror, covered in a thin mylar sheet to avoid contamination of the mercury. Credit: Aryabhatta Research Institute of Observational Sciences (ARIES).

The ILMT primary mirror has a diameter of 4.0 m, and a focal length of 8.0 m. At the principal focus, it is equipped with a detector consisting of a 4000×4000 grid of pixels, each having a side length of 13 µm.

a. One advantage of using a liquid mirror is that it is a relatively cheap way to make a very large diameter telescope. Calculate its maximum theoretical angular resolution at a wavelength of 700 nm, and compare this to the actual resolution which can be achieved with this detector due to pixel size.

[3]

b. Let x be the horizontal distance from the axis of rotation, and y be the height above the level of the mercury at the centre of rotation.

(i) By considering the forces acting on a parcel of mercury on the surface of the fluid, or otherwise, find an expression for the gradient $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the surface.

(ii) Hence, find an expression for y in terms of x, the angular velocity ω, and any relevant constants.

[6]

c. (i) Show that a vertical ray of light incident onto the mirror will be reflected to meet the axis of rotation a height a above the plane $y = 0$, where a is independent of where on the mirror the beam is incident and should be determined in terms of ω and relevant constants.

(ii) Hence determine the period of rotation needed to achieve the desired focal length.

[6]

You may find the following double angle identity useful:

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

d. (i) Over its lifetime, what is the maximum total percentage of the sky which can be imaged by the ILMT?

(ii) How long would it take for the ILMT to be able to image the star fields in this whole area?

[5]

Qu 5. Big Bang Nucleosynthesis

According to the Big Bang Theory, the very early Universe was very hot and dense, decreasing in temperature and density as the Universe expanded. As the temperature cooled, the protons and neutrons started to form nuclei like deuterium which could undergo nuclear fusion to form helium (and some other nuclei) in a process known as Big Bang Nucleosynthesis (BBN), and much later the temperature reached the point where these ionised nuclei gain electrons and became fully neutral (known as photon decoupling).

This leads to two major predictions, which have been verified with observational evidence: the existence of the Cosmic Microwave Background from the photons emitted by electrons as they lose energy during photon decoupling, and the relative abundance of helium in primordial gas. In this question, we will explore both big pieces of evidence for the Big Bang Theory.

Cosmic Microwave Background

In 1948, Ralph Alpher (working with Robert Herman) predicted that the photons released during photon decoupling should pervade the Universe with the same blackbody distribution they had at decoupling, and that the temperature of that blackbody would scale with redshift (and hence apply both before and after decoupling) as

$$T = T_0(1+z)$$

where T_0 is the temperature of the blackbody today. This blackbody radiation would later be called the Cosmic Microwave Background (CMB).

Similarly, by considering the universe to be filled with just simple pressure-less (baryonic) matter, then the density of baryonic matter would scale with redshift as

$$\rho_b = \rho_{b,0}(1+z)^3$$

where $\rho_{b,0}$ is the average density of baryonic matter in the Universe today.

He reasoned that at the time when the helium was being formed, the temperate must have been of order $\sim 10^8 - 10^{10}$ K to allow deuterium fusion to take place and the density of baryonic matter must have been of order $10^{-3} - 10^{-2}$ kg m^{-3} as much difference from this would interfere with the observed relative abundance of helium-3 and other heavier nuclei.

a. In Alpher's original paper, he used $T \cong 6 \times 10^8$ K, $\rho_b \cong 10^{-3}$ kg m^{-3} and $\rho_{b,0} \cong 10^{-27}$ kg m^{-3}. Determine his prediction for the present-day temperature of the CMB.

[2]

Penzias & Wilson found in 1965 a microwave signal coming from all parts of the sky, which was later shown to be the CMB with a temperature of ~ 3 K, close to the prediction of Alpher. More recent very accurate measurements of it give $T_0 = 2.726$ K, and the Planck space telescope has given us some of the most precise measurements of the CMB ever taken, with their final data release from 2018. They found that the redshift of decoupling (defined as when the optical depth of the plasma becomes 1, meaning that about 37% of the photons can travel unimpeded) was $z_{dec} = 1090$.

The characteristic energy of particles at a temperature T is given by $k_B T$ where k_B is the Boltzmann constant, so we can explore energy scales at different redshifts for radiation.

b. What was the characteristic energy (in eV) of the CMB photons when emitted at z_{dec}, and hence their characteristic wavelength. Which part of the EM spectrum are they from?

[4]

Using data from the Planck mission, z_{dec} corresponds to 3.71×10^5 years after the Big Bang. At this point in the Universe's history, the Universe is matter dominated and so it can be shown that

$$1 + z \propto t^{-2/3}$$

where t is the age of the Universe since the Big Bang. At even earlier times, the Universe was radiation dominated and so the relation becomes

$$1 + z \propto t^{-1/2}$$

with Planck measuring the redshift at which the transition occurred to be $z_{rad} = 3402$.

c. Decoupling did not happen instantaneously everywhere in the Universe. Instead, measurements from the WMAP satellite suggest it lasted $\Delta z = 195$. If z_{dec} corresponds to halfway through this redshift interval (so the Universe is fully matter-dominated throughout the process), calculate the time over which decoupling took place, giving your answer in years.

[3]

d. What was the temperature, characteristic energy (in eV) and age (in years) of the Universe at z_{rad}?

[2]

Relative abundance of helium

Whilst it was well understood by the 1940s how stellar nucleosynthesis could create heavy elements, it was difficult to reconcile the relative abundance of helium-4 in the Universe with what would be predicted to come from stars – explaining this was a major success of BBN and seen as the final nail in the coffin for the Steady State Theory (which relied upon stars to produce the helium).

In the very early Universe (when $z >> z_{rad}$), the temperature was high enough that neutrons turned into protons and vice versa through interactions with electrons and neutrinos, with the relative number being determined by the Boltzmann factor given their different rest mass energies:

$$\frac{N_n}{N_p} = \exp\left[-\frac{(m_n - m_p)c^2}{k_B T}\right]$$

where N_n is the number of neutrons, N_p is the number of protons, m_n is the mass of the neutron, and m_p is the mass of the proton. The ratio is "frozen out" when the characteristic energy becomes

$$k_B T_{freeze} = [(m_n - m_p) - m_e]c^2$$

where m_e is the mass of an electron. At this point, protons are still too energetic to form nuclei, and so the free neutrons decay into protons with a half life of 611 s. Eventually the temperature drops to $T_{BBN} \sim 8 \times 10^8$ K, at which point deuterium is able to form rapidly and the rest of BBN can happen, with effectively all of the neutrons (that haven't decayed by then) ending up stable in helium-4. Upon the conclusion of BBN, the relative abundance of helium-4 (by mass) can be expressed as

$$Y_{He} = \frac{M_{He}}{M_{tot}} \; .$$

e. Given the information above, calculate what the Big Bang Theory predicts for a value for Y_{He}.

[9]

Rest mass energies:
proton $m_p c^2 = 938.272$ *MeV*
neutron $m_n c^2 = 939.565$ *MeV*
electron $m_e c^2 = 0.511$ *MeV*

Qu 6. Observational Astronomy

Below is a map of the sky from Mumbai, India on an unknown date. You may draw on it to help you answer the questions, although only what is on your loose paper will be marked.

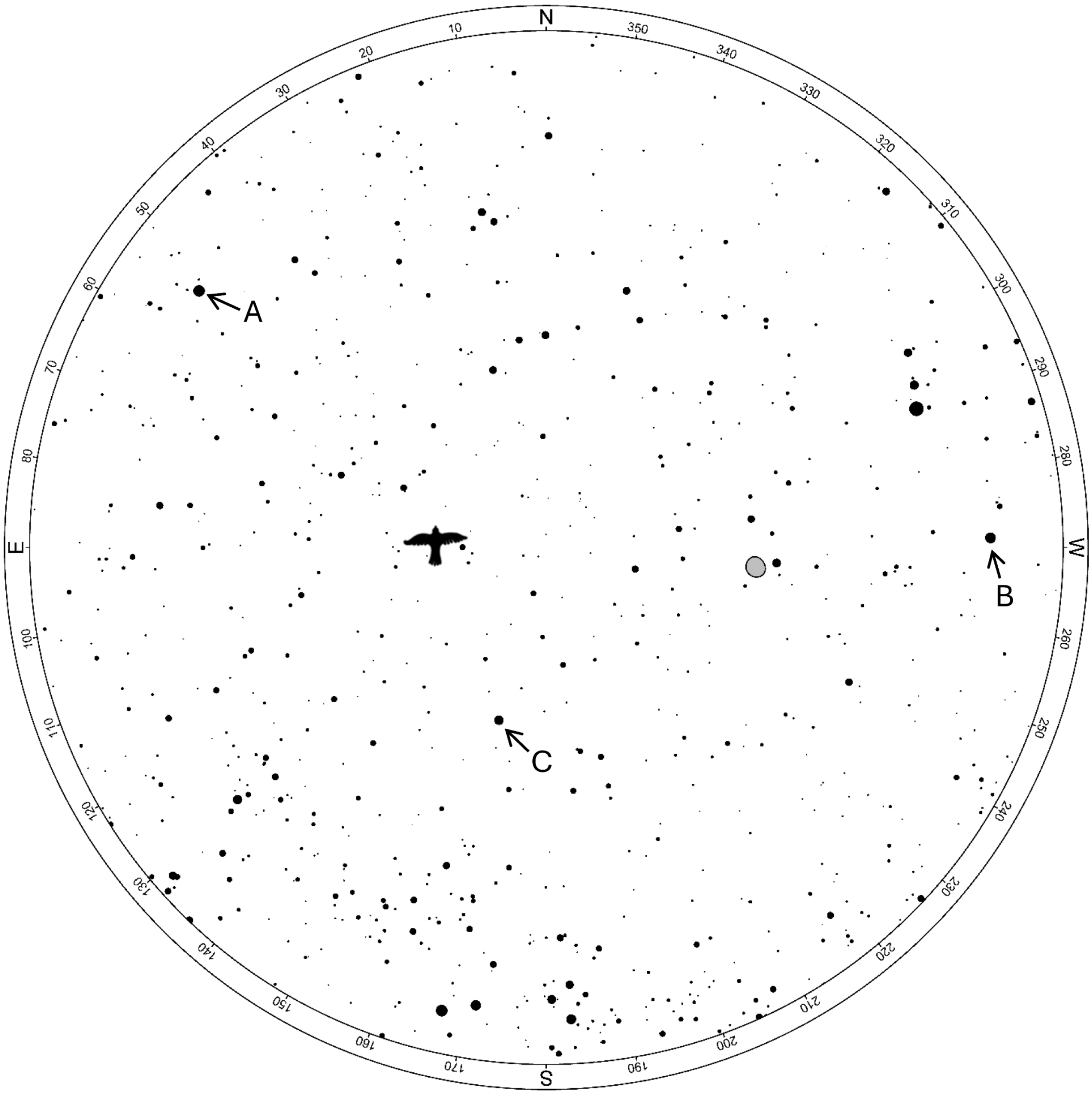


Figure 2: Sky map for use with Question 7. Altitude increases linearly from the horizon to the zenith in this figure. Stars to magnitude +4.7 are shown.

a. Name the star being pointed at with the arrows labelled A, B and C.

[3]

b. A bird was flying through the sky when you looked up and blocked a bright star from view. Name the star that is obscured by the bird.

[1]

c. Estimate the latitude of Mumbai to the nearest 2°.

[2]

d. The planet Mars is visible in this sky. State the zodiacal constellation it is in.

[1]

e. The Moon is indicated as a grey circle in the West. Which zodiacal constellation is it in?

[1]

f. Name three fully visible (i.e. fully above the horizon) constellations that the celestial equator passes through in this image.

[3]

g. One constellation has had a bright star removed and one has had a bright star added. Given that both of these are **non-zodiacal** constellations, identify them, making it clear which has lost a star and which has gained a star.

[2]

h. Only **one** of the following deep sky objects is visible in the sky: M1, M31, M42, M45, M57, M76. State which one is visible and state the angle marker (from the scale around the circumference) that it is closest to.

[2]

i. Consider the asterisms Summer Triangle and Winter Triangle. For each of them, state if they are RISING, SETTING, or CIRCUMPOLAR in this image.

[2]

j. This is the sky at 05:45 at the location. Estimate the month. Credit will only be given for a well-explained answer.
The following information may be of use, depending on your chosen method:
Regulus has a right ascension of $10^h 08^m$
Mumbai has a longitude of $73°$ *E and the time zone is UTC+05:30.*

[4]

END OF PAPER

Questions proposed by:
Dr Alex Calverley (Surbiton High School)
Charlotte Stevenson (University of Oxford)
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