British Astronomy and Astrophysics Olympiad 2024-25

Astro Round 1 Section 1

Tuesday 28th January 2025

This question paper must not be photographed or taken out of the exam room

Instructions

Time: 1 hour.

Questions: A maximum of 40 marks can be awarded for Section 1 (there are 56 marks available).

Marks: Marks allocated for each question are shown in brackets on the right. Working must be shown in order to get full credit, and it is helpful to write down numerical values of any intermediate steps.

Solutions: Answers and calculations are to be written on loose paper. Students should ensure their **name** and **school** is clearly written on the **first** answer sheet and that **all** pages are numbered. A standard formula booklet with standard physical constants may be used if desired.

Clarity: Solutions must be written legibly, in black or blue pen, and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Sitting the paper: There are two options for sitting the Astro Round 1:

1. Section 1 and Section 2 may be sat in one session of 2 hours. Section 1 should be collected in after 1 hour and only then should Section 2 be given out.
2. Section 1 and Section 2 are sat in entirely separate sessions of 1 hour each.

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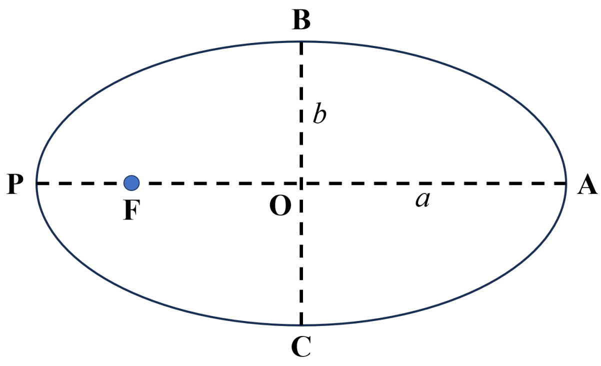
Important Constants

Constant	**Symbol**	**Value**
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Sun	$R_{\odot}$	$6.96 \times 10^{8}\,\mathrm{m}$
Radius of the Earth	$R_{\oplus}$	$6.37 \times 10^{6}\,\mathrm{m}$
Mass of the Sun	$M_{\odot}$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$M_{\oplus}$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_{\odot}$	$3.83 \times 10^{26}\,\mathrm{W}$
Absolute magnitude of the Sun	$\mathcal{M}_{\odot}$	4.74
Hubble constant	H_0	70 km s^{-1} Mpc^{-1}
Stephan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k_{B}	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck's constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Proton rest mass	m_{p}	1.67×10^{-27} kg
Electron rest mass	m_{e}	9.11×10^{-31} kg
Wien's displacement law	$\lambda_{\mathrm{max}}T$	2.90×10^{-3} m K
Avagadro's constant	N_A	6.02×10^{23} mol^{-1}

Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from $\mathbf{F}$)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from $\mathbf{F}$)
πab	area of the ellipse

Kepler's Third Law:

$$T^2 = \frac{4\pi^2}{GM}a^3$$

Vis-Viva Equation:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Wien's Displacement Law:

$$\lambda_{\max}T = \text{constant}$$

Stephan-Boltzmann Law:

$$L = 4\pi R^2\sigma T^4$$

Brightness (Intensity):

$$b = \frac{L}{4\pi d^2}$$

Magnitudes:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

$$m - \mathcal{M} = 5\log\left(\frac{d}{10}\right)$$

Distance-Parallax Relation:

$$d = \frac{1}{p}$$

Rayleigh Criterion:

$$\theta = \frac{1.22\lambda}{D}$$

Redshift:

$$z = \frac{\Delta\lambda}{\lambda_{\text{emit}}} \approx \frac{v}{c}$$

Hubble's Law:

$$v = H_0 d$$

Qu 1. Short Questions

Answer as many questions as you in can in whatever order you desire. Any mark will be capped at a maximum mark of 40 from the 56 available. Note that $\mathcal{M}_\odot$ is given in the table of constants.

A. What would be the angular diameter of the Sun if viewed from so far away that its apparent magnitude was equal to that of the brightest star in the sky, Sirius, for which $m = -1.46$. Give your answer in milliarcseconds (mas).

[2]

B. An astronomer on an island at the equator accidentally falls into a cylindrical pit, which is 2 m in diameter and 3 m deep, on their walk home during sunrise after a late night observing. Just as the Sun's rays first reach the bottom of the pit, another person walks past and sees the astronomer's predicament. They promise they will try and get help before the bottom of pit is again fully in shadow. How long do they have? Give your answer in hours and minutes, rounded to the nearest minute. (Assume the Sun is a point source and passes from East to West via directly overhead.)

[3]

C. The Parker Solar Probe passed within 3.83 million miles of the surface of the Sun during its perihelion of its most recent orbit on Christmas Eve 2024. As it did it became the fastest human-made object ever, reaching a maximum speed of $427\,000$ miles per hour. What is its orbital period in days? (1 mile = 1609 m)

[3]

D. In order to stay warm whilst doing some solar observation, an astronomer wants to light a fire with their circular pocket lens of area A and some thin scraps of wood. The lens has focal length $f = 20$ cm, and the wood has surface density $\rho = 4.2$ kg m^{-2}, specific heat capacity $c = 1360$ J kg^{-1} K^{-1}, and ignition temperature $T = 300$ °C. To minimise the effect of heat loss in the system, the astronomer wants to light the wood in $t = 5$ s. What is the minimum distance d that the astronomer should hold the wood from the lens in order to start a fire? Take the intensity of the Sun at the Earth to be $b_\odot = 1366$ W m^{-2} and the temperature of the surroundings to be $T_s = 20$ °C.

[4]

E. A sunset from a location in the Northern hemisphere takes 316 s. Given this was on a day of the year when the Sun was in the Earth's equatorial plane, deduce the latitude of the location. Ignore any effects of refraction close to the horizon.

[5]

F. A photon is emitted with an energy of 3.13 eV in a galaxy that is 365 Mpc away. What is the energy of the photon (in eV) when we receive it?

[5]

G. The bright star Formalhaut has a parallax of 0.13 arcseconds and an apparent magnitude of 1.16. Given its spectrum peaks at 338 nm, what is the radius of the star (in solar radii)?

[5]

H. A comet moves around the Sun in a generic elliptical orbit with the equation

$$\frac{(x-ae)^2}{a^2}+\frac{y^2}{b^2}=1$$

in a co-ordinate system with the Sun at the origin (corresponding to the focus F_1) and the x-axis and y-axis parallel to the major and minor axes, respectively.

(i) Show that the gradient of the ellipse at the point Y_1, which is the positive y-intercept of the ellipse, can be expressed purely in terms of the eccentricity.

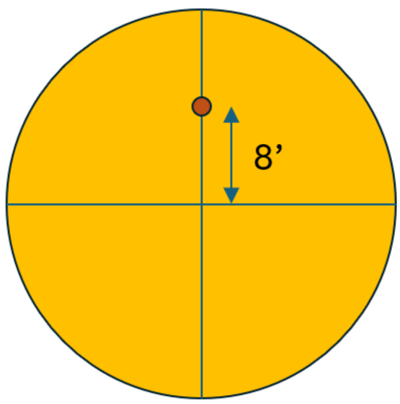
(ii) A second focus for the ellipse is at point F_2. Calculate the total length $F_1Y_1 + Y_1F_2$.

[5]

[Hint: If you have not come across implicit differentiation before, if you have $f(x, y)$ and you are differentiating with respect to x, differentiate any terms in x as you normally would and differentiate any terms in y as though they are with respect to y, but multiply the result by $\frac{\mathrm{d}y}{\mathrm{d}x}$. For example $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x+4x^2+5y^2+2y^3\right)=3+8x+10y\frac{\mathrm{d}y}{\mathrm{d}x}+6y^2\frac{\mathrm{d}y}{\mathrm{d}x}$]

I. An astronomer sees a sunspot on the Sun that has its centre an angle of 8 arcminutes directly above the centre of the solar disc (taken to be 16 arcminutes in radius on this day). It appears circular with an angular diameter of 52.7 arcseconds, although the astronomer knows that this is because its vertical height has been shortened due to the geometry of the situation. Calculate the physical area of the sunspot, giving your answer in units of the surface area of the Earth.

[7]



J. Black holes must have entropy in order to avoid having objects that enter them violate the second law of thermodynamics. In 1973, Stephen Hawking developed a relationship originally formulated by Jacob Bekenstein to give the entropy S_{BH} of a black hole as

$$S_{BH} = \frac{k_B A}{4L_P^2}$$

where A is the surface area of the event horizon (assumed to be a sphere for a non-spinning black hole), k_B is the Boltzmann constant, and L_P is the Planck length.

(i) Given the Planck length can be expressed purely in terms of the reduced Planck constant, $\hbar \equiv \frac{h}{2\pi}$, the gravitational constant, G, and the speed of light, c, in the form $L_P = \hbar^\alpha G^\beta c^\gamma$, use dimensional analysis to work out α, β, and γ.

(ii) Hence, calculate S_{BH} for our galaxy's black hole, Sagittarius A*, which has a mass of $4.31 \times 10^6\ M_\odot$.

[7]

K. Two planets are orbiting a star in circular orbits in the same direction. Planet A has a period of 120 days, whilst Planet B has a period of 200 days. The star has an angular diameter of 1° as viewed from Planet B. Planet A is seen to transit the star by observers on Planet B. Assume the radius of Planet A is negligible compared to the star, and that its transit path moves across a diameter of the stellar disc.

(i) What is the time period between transits?

(ii) Assuming the planets' orbits are coplanar, what is the duration of the transit (in hours)?

(iii) Instead Planet A's orbit is inclined at an angle of 35° to Planet B's orbit. State whether this will make the transit longer or shorter compared to the coplanar case, and calculate the size of the difference (in hours).

[10]

END OF PAPER

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