British Astronomy and Astrophysics Olympiad 2023-2024

Astro Round 1 Section 2 (Mock paper)

January 2024

This question paper must not be photographed or taken out of the exam room

Instructions

Time: 1 hour.

Questions: Answer any **TWO** questions, worth 20 marks each to give a total of 40 marks for this section (there are THREE questions available). [The real paper will have four or five questions available to allow more choice.]

Solutions: Answers and calculations are to be written on loose paper. Students should ensure their **name** and **school** is clearly written on the **first** answer sheet and that **all** pages are numbered. A standard formula booklet with standard physical constants may be used if desired.

Clarity: Solutions must be written legibly, in black pen, and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Sitting the paper: There are two options for sitting the Astro Round 1,

1. Section 1 and Section 2 may be sat in one session of 2 hours. Section 1 should be collected in after 1 hour and only then should Section 2 be given out.
2. Section 1 and Section 2 are sat in entirely separate sessions of 1 hour each.

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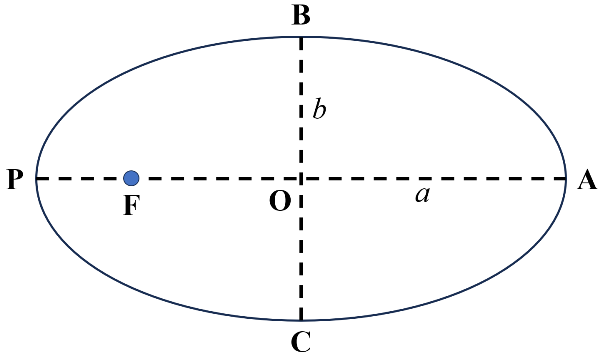
Important Constants

Constant	**Symbol**	**Value**
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Sun	$R_{\odot}$	$6.96 \times 10^{8}\,\mathrm{m}$
Radius of the Earth	$R_{\oplus}$	$6.37 \times 10^{6}\,\mathrm{m}$
Mass of the Sun	$M_{\odot}$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$M_{\oplus}$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_{\odot}$	$3.83 \times 10^{26}\,\mathrm{W}$
Absolute magnitude of the Sun	$\mathcal{M}_{\odot}$	4.74
Hubble constant	H_0	70 km s^{-1} Mpc^{-1}
Stephan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k_{B}	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck's constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Proton rest mass	m_{p}	1.67×10^{-27} kg
Electron rest mass	m_{e}	9.11×10^{-31} kg
Wien's displacement law	$\lambda_{\mathrm{max}}T$	2.90×10^{-3} m K
Avagadro's constant	N_A	6.02×10^{23} mol^{-1}

Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}$ $(= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}$ $(= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from $\mathbf{F}$)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from $\mathbf{F}$)
πab	area of the ellipse

Kepler's Third Law:

$$T^2 = \frac{4\pi^2}{GM} a^3$$

Vis-Viva Equation:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Wien's Displacement Law:

$$\lambda_{\max} T = \text{constant}$$

Stephan-Boltzmann Law:

$$L = 4\pi R^2 \sigma T^4$$

Brightness (Intensity):

$$b = \frac{L}{4\pi d^2}$$

Magnitudes:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

$$m - \mathcal{M} = 5\log\left(\frac{d}{10}\right)$$

Distance-Parallax Relation:

$$d = \frac{1}{p}$$

Rayleigh Criterion:

$$\theta = \frac{1.22\lambda}{D}$$

Redshift:

$$z = \frac{\Delta\lambda}{\lambda_{\text{emit}}} \approx \frac{v}{c}$$

Hubble's Law:

$$v = H_0 d$$

Qu 2. DART Mission

For the real paper there will be more questions to choose from - for this mock paper there are three and you should attempt only two.

The Double Asteroid Redirection Test (DART) was a mission by NASA to measure the effectiveness of the deflection of an asteroid through a high-speed impact. Such a strategy would be a cheap way of achieving planetary defence against such asteroids should one of comparable size threaten the Earth. The DART probe successfully impacted into Dimorphos, an asteroid moon of the asteroid Didymos, on 26th September 2022 (see Figure 1). The target was chosen as it would only affect the orbit of the moon and so there was no danger of it being pushed into an Earth-crossing orbit, and Dimorphos is comparable to the 140 m minimum size to be considered a potentially hazardous asteroid, so is like the real threats we might face.

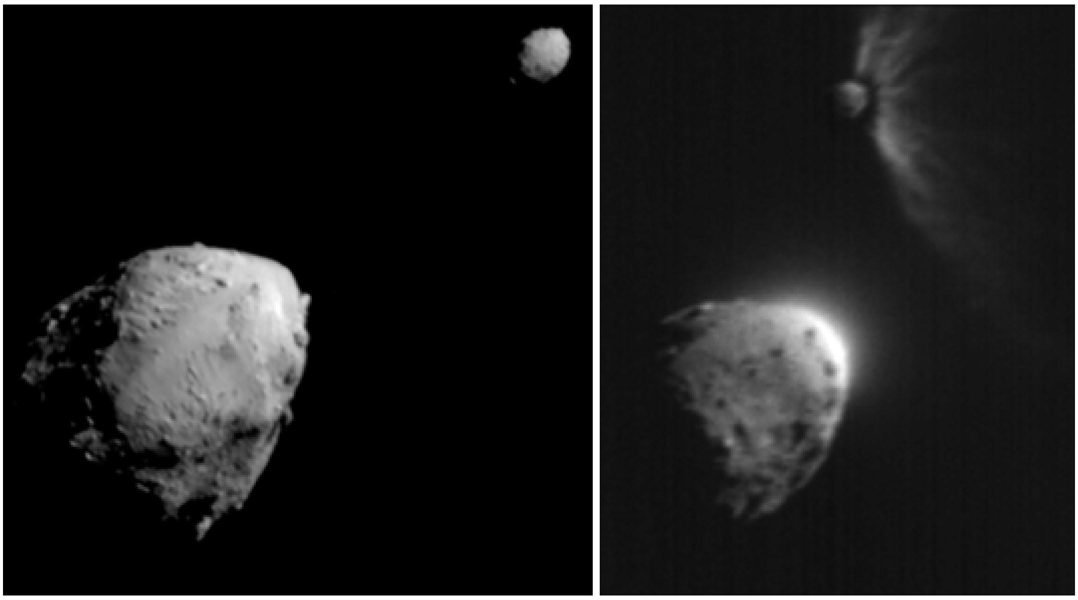


Figure 1: *Left:* The view of the asteroid Didymos (bottom left) and its moon Dimorphos (top right) taken by DART as it was heading towards the moon for impact. Credit: NASA / Johns Hopkins APL.
Right: The view of the pair taken with the probe LICIACube just after its closest approach, from a vantage point a little behind the DART impactor. There is clearly a considerable amount of material ejected from the surface of Dimorphos, as expected from a successful impact. This amplifies the effect of the impact on the orbit of Dimorphos, giving a larger change in orbital velocity than by just the probe alone. Credit: ASI / NASA.

a. Dimorphos was discovered through dips in the brightness of Didymos as it passes in front and behind the asteroid (known as transits). Radar observations of Didymos measured its diameter to be 780 m (assuming a spherical shape) and the centre-to-centre distance to Dimorphos to be 1.20 km (assuming a circular orbit). The relative depth of the dips in brightness meant that the diameter of Dimorphos was derived to be 164 m (again assuming a spherical shape), and the period of the orbital motion was 11 h 55 min.

   (i) Calculate the total mass of the binary system.

   (ii) Assuming both asteroid and moon have the same density, calculate the mass of Didymos and Dimorphos.

[3]

b. The DART probe had a mass of 570 kg at impact and was travelling at a speed of 6.14 km s^{-1} (relative to the rest frame of Dimorphos).

(i) By conserving momentum and assuming a head-on inelastic collision, determine the change in Dimorphos' orbital speed, Δv. Give your answer in mm s^{-1}.

(ii) The impact will have made the orbit elliptical, although by considering the total energy of the orbit you can calculate the radius of an equivalent circular one. Using this approach, calculate the expected decrease in orbital radius, Δr.

(iii) Hence, calculate the expected decrease in orbital period, ΔT, giving your answer in minutes.

[11]

c. Less than a month after the impact, it was announced that the observed orbital period shortening was 32 minutes. This is larger than the one in part c. due to the momentum transfer enhancement factor, β, which quantifies how much the ejecta from the surface and deformation of Dimorphos has amplified the effect. To calculate β accurately is quite complicated and will need months of observations, but a very rough estimate can be made using the approximation $\beta \propto \Delta v$. Treating the value in part c. as the $\beta = 1$ case, find the observed value of Δv and hence a ballpark estimate for the value of β.

[6]

Qu 3. Stellar Processes

The James Webb Space Telescope (JWST) has only been publicly releasing images since July 2022 and has already shown it is a more than worthy successor to the Hubble Space Telescope (HST). In Figure 2 below is a recent side by side comparison is of the 'Pillars of Creation' in the Eagle Nebula - a famous area undergoing considerable star formation.

Figure 2: The 'Pillars of Creation' - a famous star-forming region in the Eagle Nebula - as viewed with the Hubble Space Telescope (left) and the James Webb Space Telescope (right). Since the JWST operates at infrared wavelengths it can see through the dust and reveal so many of the new stars that have been born. Credit: NASA, ESA, CSA, STScI; Joseph DePasquale (STScI), Anton M. Koekemoer (STScI), Alyssa Pagan (STScI).

Some of the new stars being born will be large and hot, so will have strong stellar winds and considerable ionising radiation that will carve out holes around them, meaning that over time the dust columns as we see them will disappear. The ionised bubble is known as a Strömgren Sphere and is created by ionising photons (those with energies greater than 13.6 eV) ionising any hydrogen in the vicinity. At the edge there is a balance between new ionisations and electrons recombining with hydrogen nuclei.

The formula for the radius of the sphere is

$$r_S = \left(\frac{3N}{4\pi\alpha}\right)^{1/3} n_H^{-2/3} ,$$

where N is the number of ionising photons per second, α quantifies the probability that a proton and electron recombine into a hydrogen atom, and n_H is the number density of hydrogen nuclei in the sphere (assumed to be the same as the number density of electrons).

a. Infrared measurements by JWST suggest that in this region some O and B class stars will be formed. An example of such a star has a mass of 23 $M_\odot$, radius of 10.0 $R_\odot$, luminosity of 1.47×10^5 $L_\odot$, and effective surface temperature of 35 800 K.

 (i) Verify that photons emitted at the star's peak wavelength will be ionising.

 (ii) In the Pillars of Creation, $n_H \sim 10^9$ m^{-3} and ionisation will lead to an equilibrium temperature inside the sphere of 8000 K for which $\alpha = 3.1 \times 10^{-19}$ m^3 s^{-1}. Assuming that the star emits monochromatically at just the peak wavelength, estimate N and hence estimate r_S.

[4]

As well as the ionising effects on its surroundings, a new, bright star will exert radiation pressure, and this can trigger further collapses of regions of the nebula into other stars. Assuming the region outside the Strömgren Sphere is purely neutral hydrogen, the mass that will be collapsed by an external pressure p_0 is given as

$$M_{\text{collapse}} = \frac{1.18 k_B^2 T_{\text{neb}}^2}{p_0^{1/2} G^{3/2} m_H^2} \, .$$

This is known at the Bonner-Ebert mass, where k_B is the Boltzmann constant, T_{neb} is the temperature of the neutral hydrogen in the nebula, and m_H is the mass of a hydrogen atom. The radiation pressure is given as

$$p_{\text{rad}} = \frac{L}{4\pi r^2 c} \, ,$$

where L is the luminosity of the star, r is the distance from the star, and c is the speed of light.

b. Taking the temperature of the nebula to be $T_{\text{neb}} = 50$ K, and taking $p_0 = p_{\text{rad}}$, calculate the cloud mass that will begin to collapse just outside the Strömgren Sphere and form a new star (cluster). Give your answer in solar masses.

[2]

Radiation pressure can also be important in some stars in keeping them in equilibrium against the gravitational forces trying to compress them. The gravitational pressure at the centre of a star is given by

$$p_{\text{Grav}} = \int_0^{R_{\text{star}}} \frac{GM(r)\rho(r)}{r^2} \, \mathrm{d}r \, ,$$

where $M(r)$ is the mass enclosed within radius r, and $\rho(r)$ is the density. Since in real stars both M and ρ are complicated functions of r, this integral is not straightforward to do and generally can only be done numerically. In equilibrium, this will be balanced by contributions from the gas pressure and blackbody radiation pressure

$$p_{\text{tot}} = p_{\text{gas}} + p_{\text{rad}} = \frac{\rho_c k_B T_c}{\bar{m}} + \frac{4\sigma}{3c} T_c^4 \, ,$$

where ρ_c and T_c are the central density and central temperature, respectively, σ is the Stephan-Boltzmann constant, and $\bar{m}$ is the average mass of the particles. Generally, in the core of the star is a mix of fully ionised hydrogen and helium, so

$$\frac{\bar{m}}{m_H} = \frac{2}{1 + 3X + 0.5Y}$$

where X and Y are the mass fractions of hydrogen and helium, respectively.

c. At the centre of the Sun, lots of its initial hydrogen has already been turned into helium, so $X = 0.35$, $Y = 0.65$, $\rho_c = 1.53 \times 10^5$ kg m^{-3} and $T_c = 1.57 \times 10^7$ K.

(i) Estimate the pressure at the centre of the Sun, p_{tot}.

(ii) Calculate p_{rad} as a percentage of p_{tot}. Comment on your answer.

(iii) Consider a solar mass, Earth-sized white dwarf and assume it has a constant density equal to its average density. Carry out the integration and estimate p_{Grav}. How many times larger is it than the solar p_{tot}?

[10]

The photons generated by the nuclear fusion in the core of the Sun are gamma ray photons, however by the time they reach the surface of the Sun they are mostly visible light. This is because the photons spend a very long time being absorbed and then re-emitted by particles in the plasma, and initially this is via Compton scattering, which lowers the energy of the photon with each emission, and then later via Thomson scattering, which does not affect the photon energy but does mean it continues to bounce around in the star.

Throughout all the scattering, the photons follow a 'random walk', meaning that after N steps of average step size ℓ (known as the mean free path), the total displacement, d, is

$$d = \ell\sqrt{N}$$

d. The energy density of blackbody photons is $\frac{E}{V} = \frac{4\sigma}{c}T^4$ and the mean temperature of the Sun is 4.73×10^6 K.

   (i) By considering the total energy stored as EM radiation in the Sun and the current luminosity, estimate the time it takes for a photon to diffuse from the core to the surface. Give your answer in years.

   (ii) Hence find the mean free path, ℓ, and the number of interactions, N.

[4]

Qu 4. Observational Astronomy

Below is a map of the sky from an unknown location. You may draw on it to help you answer the questions, although only what is on your loose paper will be marked.

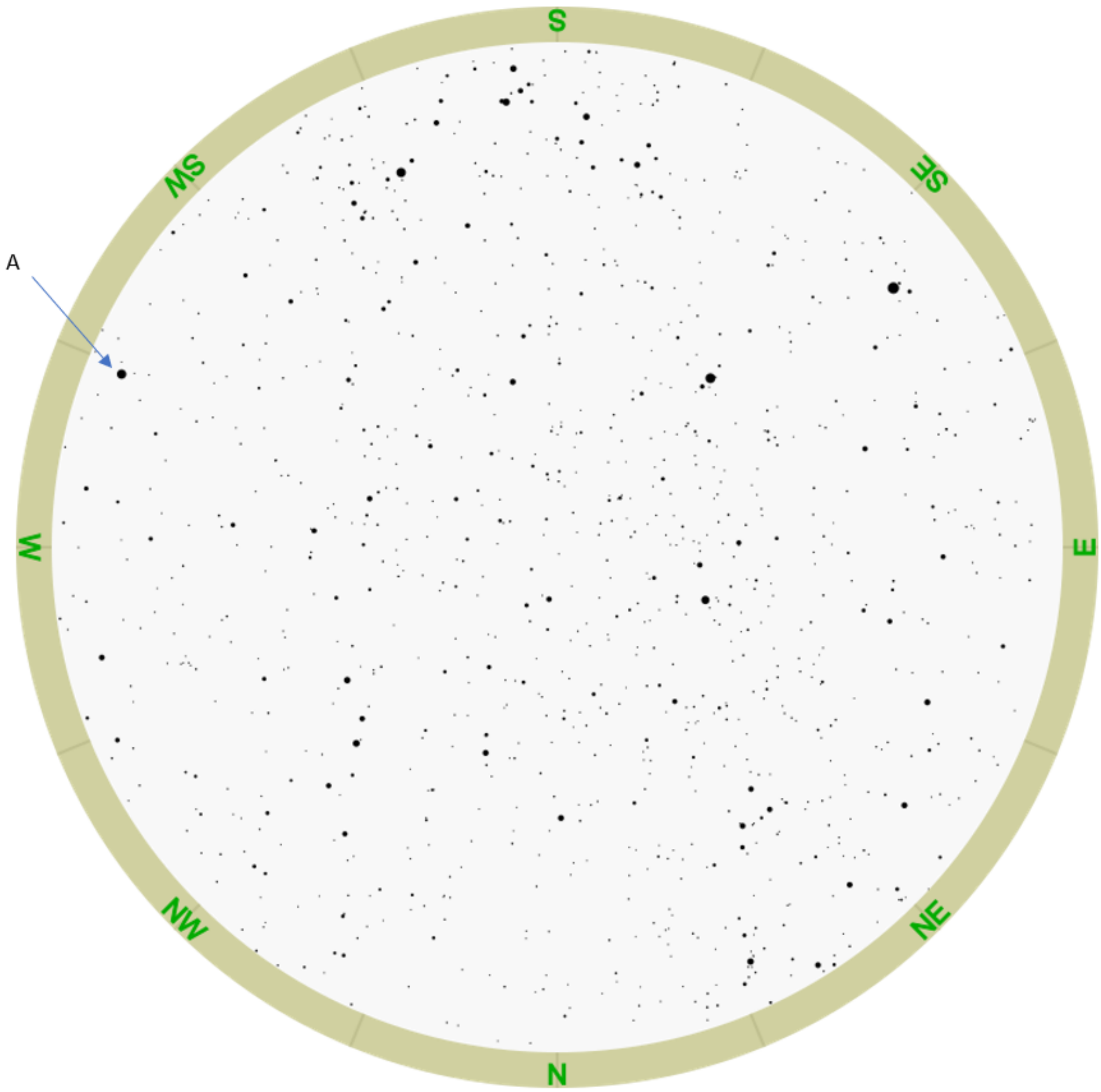


Figure 3: Sky map for use with Question 4. Altitude increases linearly from the horizon to the zenith in this figure. Stars to magnitude +4.7 are shown.

a. Name the star being pointed at with the arrow labelled A.

[1]

b. Estimate the latitude of this location to the nearest 2°.

[2]

c. Three stars have been removed. Give their names or their Bayer designation.

[3]

d. Name FOUR constellations that the ecliptic passes through in this image.

[4]

e. One planet is visible. Name the constellation it is in.

[1]

f. Consider the following list of Messier objects: M1, M31, M42, M44, M45, M57.

   (i) State which ones are above the horizon in this image.

   (ii) Which one of those is closest to the galactic equator?

   (iii) Estimate the altitude of one that is a galaxy, and its nearest compass direction on the map (from the options in the outer ring).

[5]

g. This is the sky at 21:30 at the location. Estimate the month. Credit will only be given for a well-explained answer.

[4]

END OF PAPER

Questions proposed by:
Dr Alex Calverley (Surbiton High School)