BAAO
British Astronomy and Astrophysics Olympiad

British Astronomy and Astrophysics Olympiad 2023-2024

Astro Round 1 Section 1 (Mock paper)

January 2024

This question paper must not be photographed or taken out of the exam room

Instructions

Time: 1 hour.

Questions: A maximum of 40 marks can be awarded for Section 1 (there are 43 marks available). [The real paper will have approximately 50 marks available to allow more choice.]

Solutions: Answers and calculations are to be written on loose paper. Students should ensure their **name** and **school** is clearly written on the **first** answer sheet and that **all** pages are numbered. A standard formula booklet with standard physical constants may be used if desired.

Clarity: Solutions must be written legibly, in black pen, and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Sitting the paper: There are two options for sitting the Astro Round 1,

1. Section 1 and Section 2 may be sat in one session of 2 hours. Section 1 should be collected in after 1 hour and only then should Section 2 be given out.
2. Section 1 and Section 2 are sat in entirely separate sessions of 1 hour each.

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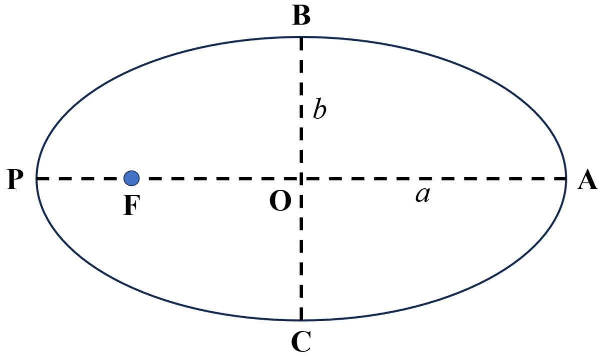
Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Sun	$R_\odot$	$6.96 \times 10^8\,\mathrm{m}$
Radius of the Earth	$R_\oplus$	$6.37 \times 10^6\,\mathrm{m}$
Mass of the Sun	$M_\odot$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$M_\oplus$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_\odot$	$3.83 \times 10^{26}\,\mathrm{W}$
Absolute magnitude of the Sun	$\mathcal{M}_\odot$	4.74
Hubble constant	H_0	70 km s^{-1} Mpc^{-1}
Stephan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann constant	k_B	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck's constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Proton rest mass	m_p	1.67×10^{-27} kg
Electron rest mass	m_e	9.11×10^{-31} kg
Wien's displacement law	$\lambda_\mathrm{max} T$	2.90×10^{-3} m K
Avagadro's constant	N_A	6.02×10^{23} mol^{-1}

Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from **F**)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from **F**)
πab	area of the ellipse

Kepler's Third Law:

$$T^2 = \frac{4\pi^2}{GM}a^3$$

Vis-Viva Equation:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Wien's Displacement Law:

$$\lambda_{\max}T = \text{constant}$$

Stephan-Boltzmann Law:

$$L = 4\pi R^2\sigma T^4$$

Brightness (Intensity):

$$b = \frac{L}{4\pi d^2}$$

Magnitudes:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

$$m - \mathcal{M} = 5\log\left(\frac{d}{10}\right)$$

Distance-Parallax Relation:

$$d = \frac{1}{p}$$

Rayleigh Criterion:

$$\theta = \frac{1.22\lambda}{D}$$

Redshift:

$$z = \frac{\Delta\lambda}{\lambda_{\text{emit}}} \approx \frac{v}{c}$$

Hubble's Law:

$$v = H_0 d$$

Qu 1. Short Questions

For the real paper there will be more questions to choose from - for this mock paper there is a maximum mark of 40 from the 43 available.

A. Io and Europa are both moons of Jupiter. Europa takes twice as long as Io to complete an orbit. What is the ratio of the centripetal acceleration of Io and Europa, $\frac{a_{\mathrm{Io}}}{a_{\mathrm{Europa}}}$?

[3]

B. A binary star system is 2140 light years away and consists of two stars like the Sun. The average separation between the stars is 0.00593 light years. Determine the telescope diameter needed to resolve them if using a visible wavelength of 550 nm.

[2]

C. The Sun currently has its peak wavelength as 500 nm. What will be the new peak wavelength when it becomes a red giant, given its radius will be 200 times larger and its luminosity 4000 times larger?

[2]

D. One spectral line in hydrogen is caused by photons with an energy of 2.55 eV. The same line is redshifted in the spectrum of a distant galaxy by 5.4 nm. How far away is the galaxy? Give your answer in megaparsecs (Mpc).

[4]

E. An asteroid in a circular orbit around the Sun is at its closest to Earth every 300 days. What is its orbital speed? Assume the Earth's orbit is also circular and that both orbit in **opposite** directions.

[3]

F. Protons have a radius of about 10^{-15} m. Assuming constant nuclear density, suggest the radius of a 1.4 $M_{\odot}$ neutron star in kilometres.

[3]

G. In some Islamic countries, the start of a new month is determined through direct telescopic observations of the Moon, looking for a very thin crescent. This is a very hard measurement to make and the human eye struggles to recognise the presence of a crescent until about 0.6% of the lunar disc is illuminated. Calculate how many hours this is after the theoretical (astronomical) New Moon. Assume the lunar orbit is circular and has a period of 29.53 days.

[5]

H. What fraction of the Earth's surface can be seen by a GPS satellite at any one time, given they have a period of half a day?

[5]

I. The James Webb Space Telescope (JWST) has now entered a halo orbit around the second Lagrangian point, L_2, of the Sun-Earth system at a distance of about 1.5 million km from Earth, directly along the Sun-Earth line. The rectangular sunshield is rather large (measuring 21 m by 14 m), very reflective (reflecting $\sim 90\%$ of the incident light), and always points directly towards the Sun to protect the other parts of the telescope. Calculate the apparent magnitude of the JWST at L_2 due to the light reflected off its sunshield, given the Sun's absolute magnitude is $\mathcal{M} = 4.74$.

[7]

J. The proton–proton chain reaction is one of two known sets of nuclear fusion reactions by which stars convert hydrogen to helium. It dominates in stars with masses less than or equal to that of the Sun, and involves a net change of four hydrogen nuclei becoming one helium nucleus.

(i) How much energy is produced in this reaction? [Take the mass of one hydrogen nucleus as $m_H = 1.674 \times 10^{-27}$ kg, and the mass of one helium nucleus as $m_{He} = 6.649 \times 10^{-27}$ kg.]

(ii) Calculate the number of hydrogen nuclei that must fuse to form helium every second to provide the measured solar luminosity, and the percentage of hydrogen mass per reaction that is converted into energy via fusion.

(iii) The Sun is composed of about 71% hydrogen, 27% helium and some heavier elements. However, only 13% of the hydrogen is available for hydrogen fusion in the core. The rest remains in layers of the Sun where the temperature is too low for fusion to occur. Use these figures and the answers to previous questions to calculate the lifetime in years of hydrogen fusion in the Sun (called its main sequence lifetime), assuming the Sun's luminosity remains constant.

[9]

END OF PAPER

Questions proposed by:
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