British Astronomy and Astrophysics Olympiad 2022-23

Astronomy & Astrophysics Challenge Paper

September - December 2022

Instructions

Time: 1 hour (30 marks).

Questions: Answer all questions in Sections A and B, but only **one** question in Section C.

Marks: Marks allocated for each question are shown in brackets on the right. Working must be shown in order to get full credit, and it is helpful to write down numerical values of any intermediate steps.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. A standard formula booklet with standard physical constants should be supplied.

Eligibility: All sixth form students (or younger) are eligible to sit any BAAO paper.

Further Information about the British Astronomy and Astrophysics Olympiad

This is the first paper of the British Astronomy and Astrophysics Olympiad in the 2022-2023 academic year. To progress to the next stage of the BAAO, you **must** *take the BPhO Round 1 in November 2022, which is a general physics problem paper. Those achieving a Top Gold will be invited to take the BAAO Competition paper on* **Monday 23rd January 2023**, *as will those achieving a Distinction on this paper.*

To be awarded the highest grade (Distinction) in this paper, it should be sat under test conditions and marked papers achieving 60% or above should be scanned and uploaded to a folder linked to from the BPhO platform by **Friday 21st October 2022**. *All papers, including papers sat after that date, or below that mark (i.e. Merit or Participation), should have their results recorded using the online form (in the platform) by* **Friday 2nd December 2022**.

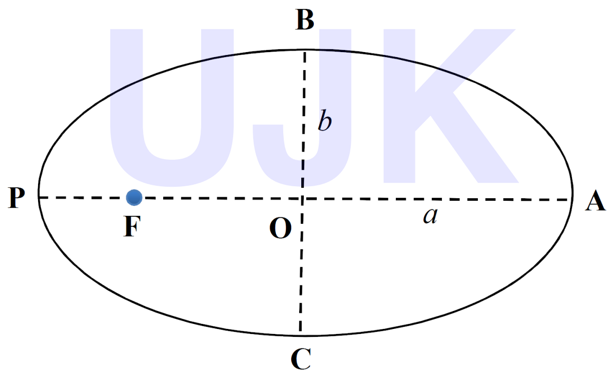
To solve some of the questions, you will need to write equations, draw diagrams and, in general, show your working. You are also encouraged to look at the clear sky and identify the brightest stars, a few days before sitting the paper.

This paper has more than an hour's worth of questions. You are encouraged to have a go at as many as you can and to follow up on those that you do not complete in the time allocated.

Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Earth	R_E	$6.37 \times 10^6\,\mathrm{m}$
Semi-major axis of the Earth's orbit		1 au
Radius of the Sun	$R_\odot$	$6.96 \times 10^8\,\mathrm{m}$
Mass of the Sun	$M_\odot$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_\odot$	$3.85 \times 10^{26}\,\mathrm{W}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from **F**)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from **F**)
πab	area of the ellipse

Kepler's Third Law: For an elliptical orbit, the square of the period, T, of an object about the focus is proportional to the cube of the semi-major axis, a (as defined above), such that

$$T^2 = \frac{4\pi^2}{GM} a^3 \,,$$

where M is the total mass of the system (typically dominated by the central object) and G is the universal gravitational constant.

Magnitudes: The apparent magnitudes of two objects, m_1 and m_0, are related to their apparent brightnesses, b_1 and b_0, via the formula:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

Section A: Multiple Choice

Write the correct answer to each question. Each question is worth 1 mark. There is only one correct answer to each question. **Total: 10 marks**.

1. Below is one of the first images taken with the James Webb Space Telescope (JWST) released to the public on 12^{th} July 2022. Which object is it from?

   A. Carina Nebula
   B. Stephan's Quintet
   C. Southern Ring Nebula
   D. WASP-96 b

2. In Autumn 2022, NASA plans to launch a practice mission in preparation for sending humans back to the Moon. What is the name of the programme this is part of?

   A. Artemis
   B. Athena
   C. Diana
   D. Orion

3. A telescope with a focal length of 750 mm is used with an eyepiece. Which eyepiece focal length would give the greatest overall magnification?

   A. 25 mm
   B. 20 mm
   C. 15 mm
   D. 10 mm

4. The Gaia spacecraft, launched in 2013, is designed to map the sky with unprecedented precision. Its third data release (from June 2022) had 1.8×10^9 objects detected of which 1.9×10^6 were highly likely to be quasars (distant galaxies with an actively feeding black hole responsible for their light output). Assuming the quasars are evenly distributed across the whole sky, what is the approximate average angular separation between them?

   A. $\sim 0.001°$
   B. $\sim 0.01°$
   C. $\sim 0.1°$
   D. $\sim 1.0°$

5. In September 2022, Jupiter reached opposition a few days after the autumnal equinox. Which constellation was it in at the time? [Jupiter's opposition corresponds to when it is closest in its orbit to the Earth.]

   A. Cancer
   B. Pisces
   C. Scorpio
   D. Virgo

6. An observer on the equator sees the Moon rise at 22:00. Ignoring the inclination and eccentricity of the Moon's orbit, when will it rise the next night? The Moon's orbital period is 27.3 days.

   A. 21:07
   B. 21:47
   C. 22:13
   D. 22:53

7. Which of these stars will culminate at the highest altitude as seen by an observer in Oxford (latitude $51.75°$ N, longitude $1.26°$ W)?

   A. Aldebaran (Right ascension $= 04^{\mathrm{h}}\ 36^{\mathrm{m}}$, declination $= +16.51°$)
   B. Altair (Right ascension $= 19^{\mathrm{h}}\ 51^{\mathrm{m}}$, declination $= +8.87°$)
   C. Capella (Right ascension $= 05^{\mathrm{h}}\ 17^{\mathrm{m}}$, declination $= +46.00°$)
   D. Procyon (Right ascension $= 07^{\mathrm{h}}\ 39^{\mathrm{m}}$, declination $= +5.22°$)

8. The cubesat CAPSTONE will enter a near-rectilinear halo orbit (NRHO) of the Moon on 13^{th} November 2022 as a direct test of the orbit planned for the Lunar Gateway - a space station due to be built in orbit around the Moon by the end of the decade. It is a polar orbit, going from 1500 km above the North pole to $70\,000$ km above the South pole. Treating it as an ellipse, what is the eccentricity of the orbit? The Moon has a radius of 1740 km.

   A. 0.83
   B. 0.88
   C. 0.91
   D. 0.96

9. An asteroid in a circular orbit around the Sun is at its closest to Earth every 300 days. What is its orbital speed? Assume the Earth's orbit is also circular and that both orbit in the same direction.

   A. 21 km s^{-1}
   B. 28 km s^{-1}
   C. 32 km s^{-1}
   D. 39 km s^{-1}

10. Jupiter's apparent magnitude at opposition in 2022 of $m = -2.94$ is the brightest for ~ 70 years due to it happening close to Jupiter's perihelion. What is the difference in apparent magnitude between Jupiter's brightest and faintest possible oppositions? Jupiter has a semi-major axis of 5.20 au and an eccentricity of 0.0489. Assume the Earth's orbit is circular.

    A. 0.38
    B. 0.43
    C. 0.48
    D. 0.53

Section B: Short Answer

Each short question is worth 5 marks. **Total: 10 marks**.

Shepherd Moon of Neptune

11. Although Saturn is famous for its rings, all of the gas giants in the Solar System have ring systems. The outer ring is known as the Adams ring and is very thin. Normally such a thin structure would widen over time so there needs to be a process keeping it constrained. One hypothesis is that the Neptunian moon Galatea, with an orbit just slightly smaller than the ring, acts as a 'shepherd moon' by having a $42 : 43$ orbital resonance with particles in the ring, in terms of the period of their orbits. The ring and the moon are shown in Figure 1.

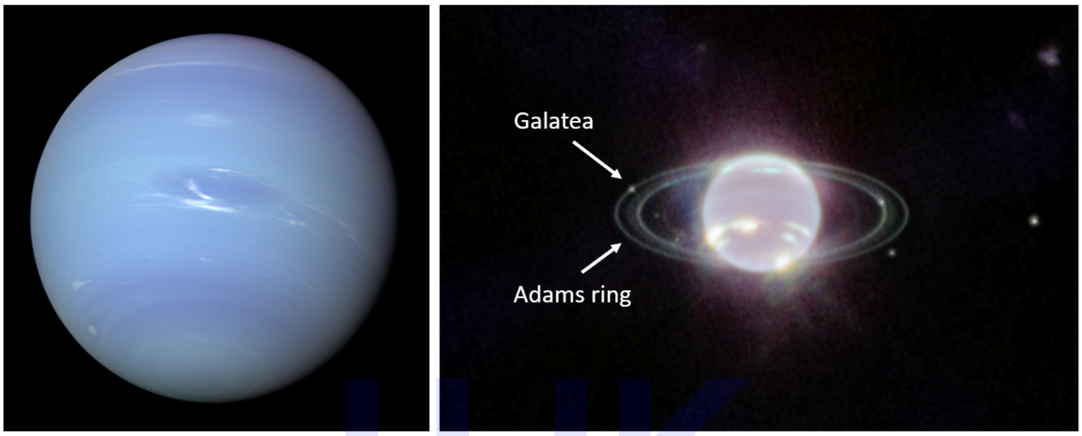


Figure 1: *Left:* Neptune as seen by the Voyager 2 mission in August 1989, a few days before its flyby. Credit: NASA / JPL / Voyager-ISS / Justin Cowart.
Right: Neptune and its ring system as imaged in the infrared by the NIRCam instrument on the James Webb Space Telescope in July 2022. Multiple moons and rings are visible, with Galatea and the Adams ring labelled. Credit: NASA / ESA / CSA / STScI / Joseph DePasquale.

The semi-major axis of Galatea is 61953 km. Assume the moon and the ring particles travel in circular orbits.

(a) Calculate the semi-major axis of a particle in the centre of the ring.

[3]

(b) Measurements of the amplitude of ripples in the ring caused by gravitational attraction towards the moon indicate the ring particles closest to the moon have a radial acceleration of 0.15 mm s^{-1}. Estimate the mass of the moon.

[2]

Distance to an Ancient Galaxy

12. The very first image released by the James Webb Space Telescope (JWST) was of a galaxy cluster called SMACS 0723. The image is considered to be Webb's first deep field, since a long exposure time of 12.5 hours was used to allow the light from very faint and distant galaxies to be seen. The spectrum of one such galaxy is shown in Figure 2.

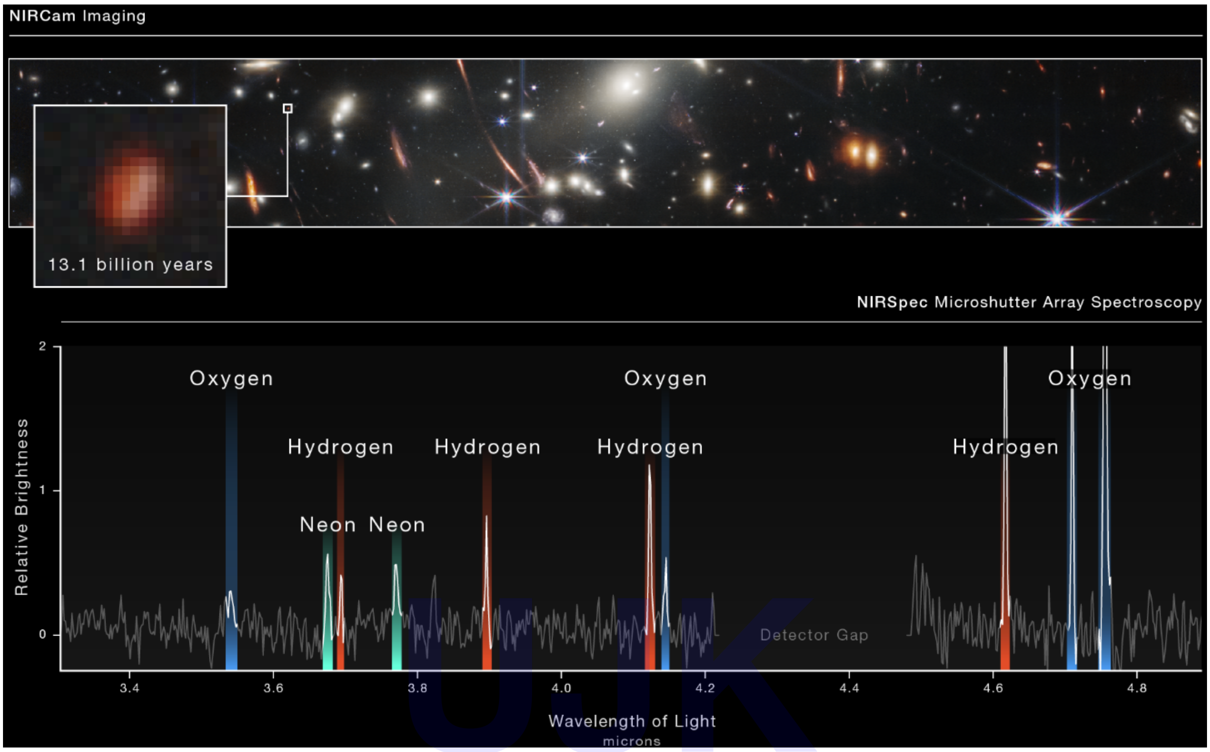


Figure 2: Highly redshifted emission lines in the spectrum of a galaxy that is 13.1 billion years old, captured using the JWST's near-infrared spectrometer (NIRSpec). Credit: NASA, ESA, CSA, STScI.

The spectrum shows four bright hydrogen lines, which are part of the Balmer series (some of which are normally seen in the visible). The rest frame wavelengths of the longest four lines in the series are 410 nm, 434 nm, 486 nm and 656 nm (**not all of which are visible in the spectrum**).

Once a redshift is known, its recessional velocity can be calculated. At very high redshifts, such as these, General Relativity must be used. A conversion from redshift to recessional velocity is shown in Figure 3.

(a) Taking measurements from the spectrum, estimate the redshift of the galaxy. [Hint: you should measure more than one line to ensure you correctly identify which rest frame wavelength corresponds to which line.]

[3]

(b) Taking the value of the Hubble constant to be $H_0 = 70$ km s^{-1} Mpc^{-1}, what is the distance to the galaxy? Give your answer in Mpc.

[2]

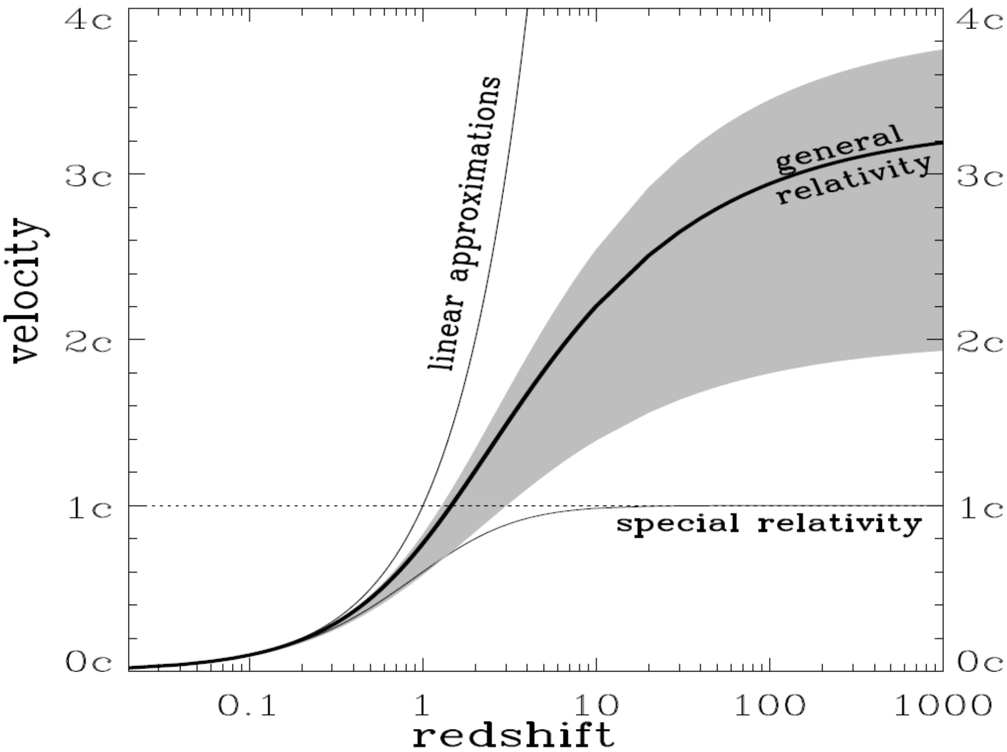


Figure 3: Conversion from redshift to recessional velocity for a linear approximation $(v = zc)$, using Special Relativity $\left(v = c\frac{(1+z)^2-1}{(1+z)^2+1}\right)$, and using General Relativity $\left(v = \dot{a}(z)\int_0^z \frac{c\,dz'}{H(z')}\right)$. The grey area corresponds to a variety of values for cosmological parameters. The solid line corresponds to values approximately the same as the current measured cosmological parameters. Credit: Davis & Lineweaver (2001).

Section C: Long Answer

Each long question is worth 10 marks. Answer **either** *Qu 13* **or** *Qu 14.* **Total: 10 marks**.

Transit by the ISS

13. Figure 4 below is a composite image which depicts a transit of the International Space Station (ISS) across the disc of the Sun. The image comprises 26 individual photographs which were taken at regular time intervals during the transit. The total duration of the transit was less than one second. In this question we will ignore any effects caused by the rotation of the Earth.

Figure 4: A composite of a selection of the frames taken with a high-speed camera of a transit of the ISS in front of the Sun, taken from Northamptonshire at 10:22 BST on 17th June 2022. Credit: Jamie Cooper Photography

(a) During the transit, the angular diameter of the Sun as viewed from the position of the camera was $31'29''$. Use the image to calculate the angle θ_1 subtended by the ISS between the first and last photographs, as viewed from the position of the camera. Note that the centre of the solar disc is NOT in the field of view of the photograph. [You are given that $1^\circ = 60' = 3600''$.]

[4]

(b) The ISS maintained a mean height of 415 km above the surface of the Earth during the transit. What is the angle θ_2 subtended by the ISS between the first and last photographs, as viewed from the centre of the Earth? [Assume that the Sun was at the zenith (directly overhead) during the transit.]

[3]

(c) The orbital period of the ISS is approximately 93 min. Estimate the frame rate of the camera used to photograph the transit.

[3]

Seeing the JWST

14. The James Webb Space Telescope (JWST) is an exciting new space-based observatory which is capable of detecting incredibly faint objects that have never been seen before, but it is also possible to be seen from Earth if you have a large enough telescope. It has now entered a halo orbit around the second Lagrangian point, L_2, of the Sun-Earth system at a distance of about 1.5 million km from Earth, directly along the Sun-Earth line.

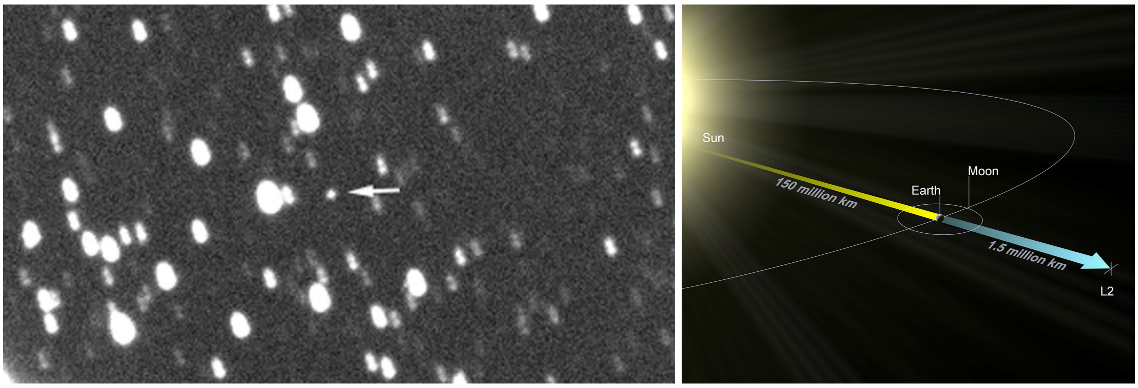


Figure 5: *Left:* An image of NASA's James Webb Space Telescope reaching its final distance from Earth. It is a tiny speck among a sea of background stars. The stars appear smudged because the telescope was tracking the motion of JWST, which appears as a small white speck. Credit: Gianluca Masi / The Virtual Telescope Project. *Right:* The position of the second Lagrangian point, L_2, relative to the Earth. Credit: ESA.

The rectangular sunshield is rather large (measuring 21 m by 14 m, roughly the same as a tennis court), very reflective (reflecting $\sim 90\%$ of the incident light), and always points directly towards the Sun to protect the other parts of the telescope, especially to keep it cool enough to do infrared astronomy.

(a) Calculate the apparent magnitude of the JWST at L_2 due to the light reflected off its sunshield, given the apparent magnitude of the Sun is $m_\odot = -26.832$ as viewed from Earth. [Hint: you may wish to calculate the intensity of light that corresponds to an apparent magnitude of zero.]

[7]

(b) On a very dark night, your eye's pupil opens up to about 6 mm in diameter and you are able to see stars as faint as $m = +6$. Estimate what diameter of telescope you would need to look through in order to see the JWST in similarly dark conditions.

[3]

END OF PAPER

Questions proposed by:
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