BAAO
British Astronomy and Astrophysics Olympiad

British Astronomy and Astrophysics Olympiad 2018-2019

Astronomy & Astrophysics Challenge Paper

September - December 2018

Instructions

Time: 1 hour.

Questions: Answer all questions in Sections A and B, but only **one** question in Section C.

Marks: Marks allocated for each question are shown in brackets on the right. Working must be shown in order to get full credit, and you may find it useful to write down numerical values of any intermediate steps.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. A standard formula booklet with standard physical constants should be supplied.

Eligibility: All sixth form students (or younger) are eligible to sit any BAAO paper.

Further Information about the British Astronomy and Astrophysics Olympiad

This is the first paper of the British Astronomy and Astrophysics Olympiad in the 2018-2019 academic year. To progress to the next stage of the BAAO, you **must** *take the BPhO Round 1 in November 2018, which is a general physics problem paper. Those achieving at least a Gold will be invited to take the BAAO Competition paper on* **Monday 21st January 2019**.

To be awarded the highest grade (Distinction) in this paper, it should be sat under test conditions and marked papers achieving 60% or above should be sent in to the BPhO Office in Oxford by **Friday 19th October 2018**. *Papers sat after that date, or below that mark (i.e. Merit or Participation), should have their results recorded using the online form by* **Friday 7th December 2018**.

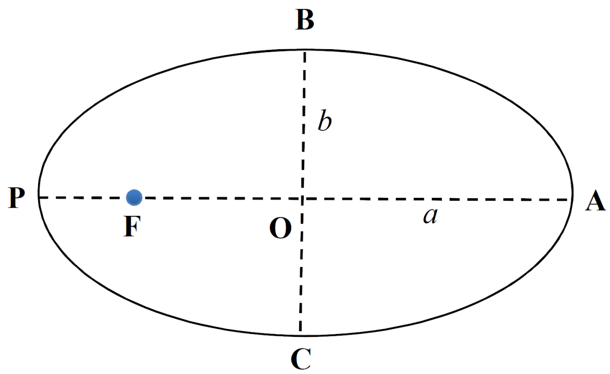
To solve some of the questions, you will need to write equations, draw diagrams and, in general, show your working. You are also encouraged to look at the clear sky and identify the brightest stars, a few days before sitting the paper.

This paper has more than an hour's worth of questions. You are encouraged to have a go at as many as you can and to follow up on those that you do not complete in the time allocated.

Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Earth	R_E	$6.37 \times 10^{6}\,\mathrm{m}$
Semi-major axis of the Earth's orbit		1 au
Radius of the Sun	$R_\odot$	$6.96 \times 10^{8}\,\mathrm{m}$
Mass of the Sun	$M_\odot$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_\odot$	$3.85 \times 10^{26}\,\mathrm{W}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
F	focus
P	periapsis (point nearest to F)
A	apoapsis (point furthest from F)

Keplers Third Law: For an elliptical orbit, the square of the period, T, of an object about the focus is proportional to the cube of the semi-major axis, a (as defined above), such that

$$T^2 = \frac{4\pi^2}{GM} a^3 \,,$$

where M is the total mass of the system (typically dominated by the central object) and G is the universal gravitational constant.

Magnitudes: The apparent magnitudes of two objects, m_1 and m_0, are related to their apparent brightnesses, b_1 and b_0, via the formula:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

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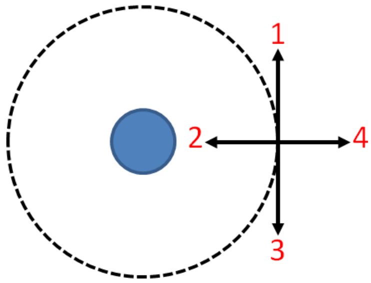
Section A: Multiple Choice

Write the correct answer to each question. Each question is worth 1 mark. There is only one correct answer to each question. **Total: 10 marks**.

1. For a satellite in a circular orbit around the Earth, which of the arrows in the figure below describe the direction of the velocity and the resultant force?



   A. Velocity $= 1$, Resultant force $= 1$

   B. Velocity $= 1$, Resultant force $= 2$

   C. Velocity $= 2$, Resultant force $= 1$

   D. Velocity $= 3$, Resultant force $= 4$

2. The tail of a comet

   A. is gas and dust pulled off the comet by the Sun's gravity

   B. always points towards the Sun

   C. is a trail behind the comet, pointing away from the Sun as the comet approaches it, and toward the Sun as the comet moves out of the inner Solar System

   D. is gas and dust expelled from the comet's nucleus and blown outward by radiation pressure and solar wind

3. Meteor showers are caused by

   A. the breakup of asteroids that hit our atmosphere at predictable times

   B. the Earth passing through the debris left behind by a comet as it moves through the inner Solar System

   C. passing asteroids triggering auroral displays

   D. nuclear reactions triggered by an abnormally large meteoritic particle entering the upper atmosphere

4. Looking up into the UK sky at 10pm in late September, which of the following bright stars is NOT visible?

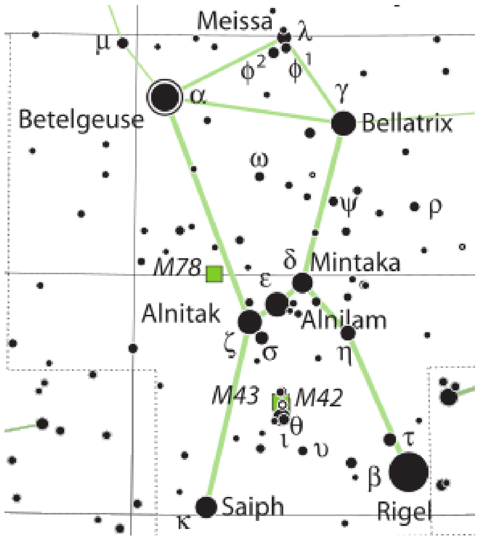
   A. Deneb (Right ascension $= 20^{\mathrm{h}}\ 41^{\mathrm{m}}$, declination $= +45.28°$)

   B. Vega (Right ascension $= 18^{\mathrm{h}}\ 37^{\mathrm{m}}$, declination $= +38.78°$)

   C. Capella (Right ascension $= 05^{\mathrm{h}}\ 17^{\mathrm{m}}$, declination $= +46.00°$)

   D. Sirius (Right ascension $= 06^{\mathrm{h}}\ 45^{\mathrm{m}}$, declination $= -16.72°$)

5. The three stars that make up Orion's belt are Alnitak, Alnilam and Mintaka, with apparent magnitudes, m, of 1.77, 1.69 and 2.23 respectively. What is the ratio of the apparent brightnesses of the two brightest stars?



A. 0.655

B. 1.076

C. 1.528

D. 1.644

6. A comet is orbiting the Sun in an orbit with a semi-major axis of 5.0 au and an eccentricity, $e = 0.80$. Calculate its semi-minor axis, b.

A. 1.0 au

B. 2.0 au

C. 3.0 au

D. 4.0 au

7. What is the period of the comet from the previous question?

A. 2.1 years

B. 2.9 years

C. 5.2 years

D. 11.2 years

8. On the evening of 27th July 2018, there was a total lunar eclipse (where the Moon passes into the Earth's shadow). As viewed from Oxford, totality began whilst the Moon was below the horizon. Interestingly, moonrise was at 8:55pm, yet sunset was only at 9:01pm, so for 6 minutes both the fully eclipsed Moon and setting Sun were visible above the horizon. This very rare event is known as a selenelion. What is the explanation behind this seemingly impossible observation?

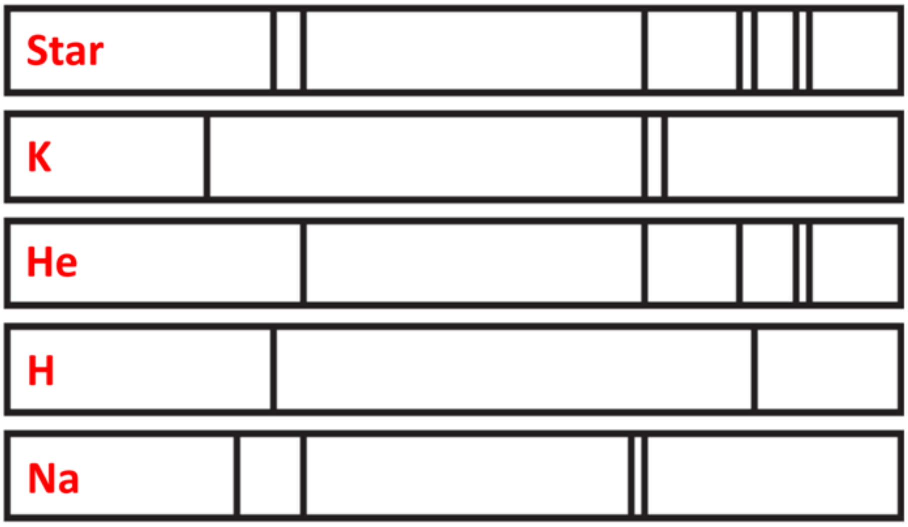
A. The Moon is in an orbit with a non-zero eccentricity

B. Prominences on the Sun at the time of the eclipse

C. The non-zero light travel time to cover the large distance between the Moon and the Earth

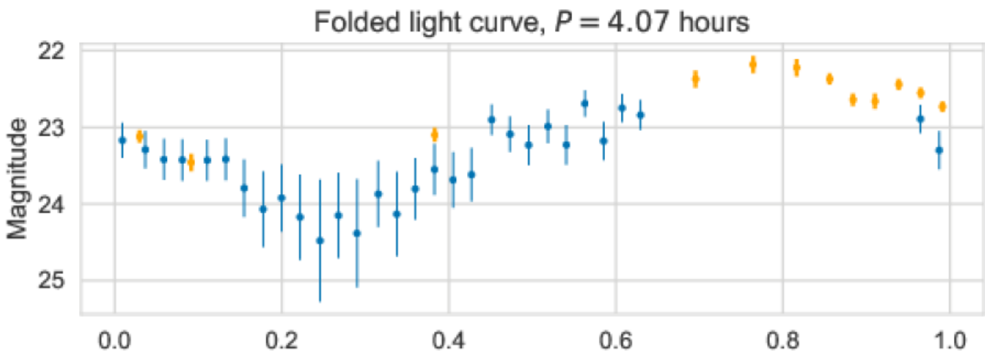
D. The effect of atmospheric refraction meaning the Sun and Moon only appear to be above the horizon

9. Light from a star is split into a line spectrum of different colours. The line spectrum from the star is shown, along with the line spectra of some individual elements. Identify the elements present in the star.



A. Helium and hydrogen
B. Potassium and sodium and hydrogen
C. Hydrogen and sodium
D. Sodium and potassium

10. 'Oumuamua is an elongated interstellar comet. During an observation with a ground-based telescope whilst close to the Sun, the light curve displayed a sinusoidal variation in magnitude as shown below. If the comet is modelled as an ellipsoid with a shortest visible axis of 30 m, use the light curve to determine the approximate size of the longest visible axis. Assume that the major axis lies in the plane along the line of sight.



A. ~ 60 m
B. ~ 240 m
C. ~ 960 m
D. ~ 3840 m

Section B: Short Answer

Each short question is worth 5 marks. **Total: 10 marks**.

Neso and Mercury

11. Neso is the outermost moon of Neptune, and is notable for being the moon with the greatest known orbital distance from its planet, corresponding to a period of over 26 Earth years. At its furthest point (apoapsis), it is further from Neptune than Mercury gets from the Sun, meaning it is close to the limit of being held by Neptune's gravity (see Figure 1).

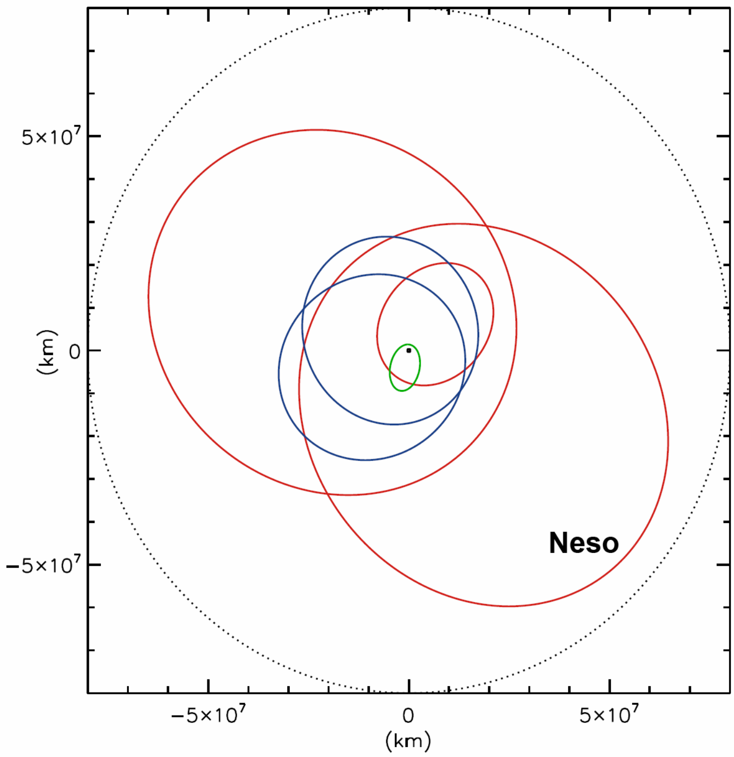


Figure 1: Plan view of Neptune's outermost satellite orbits. Neptune is represented by the black dot at the centre. The green orbit corresponds to Nereid, the two in blue are the prograde orbits of Sao and Laomedeia, and the red are the retrograde orbits of Halimede, Psamathe and Neso (bottom right). The dotted circle shows the theoretical outer limit of stability for Neptune satellites. Taken from Shepperd *et al.* (2006).

Orbital data about Neso and Mercury are summarised below:

	Mercury	Neso
Semi-major axis, a ($\times 10^9$ m)	57.909	49.285
Eccentricity, e	0.2056	0.5714
Orbital period, T (Earth days)	87.97	9740.73

(a) Given that the apoapsis of an elliptical orbit can be calculated as $r_{\rm ap} = a(1+e)$, verify that the apoapsis of Neso's orbit is greater than that of Mercury's.

[1]

(b) Use the orbital data to work out the ratio of Neptune's mass to the Sun's, $M_{\rm Nep}/\,M_{\odot}$.

[3]

(c) Hence work out the mass of Neptune, $M_{\rm Nep}$.

[1]

Cosmic Microwave Background

12. The cosmic microwave background (CMB) is measured today to have an almost perfectly uniform temperature of $T_0 = 2.725$ K. The temperature of the CMB at any redshift can be calculated using $T = T_0(1+z)$ where z is the redshift. During the early expansion of the Universe, the temperature was high enough to ionize the hydrogen atoms filling the universe and make the universe opaque. Once the temperature dropped below 3000 K the universe became transparent and the CMB was emitted in a process known as photon decoupling.

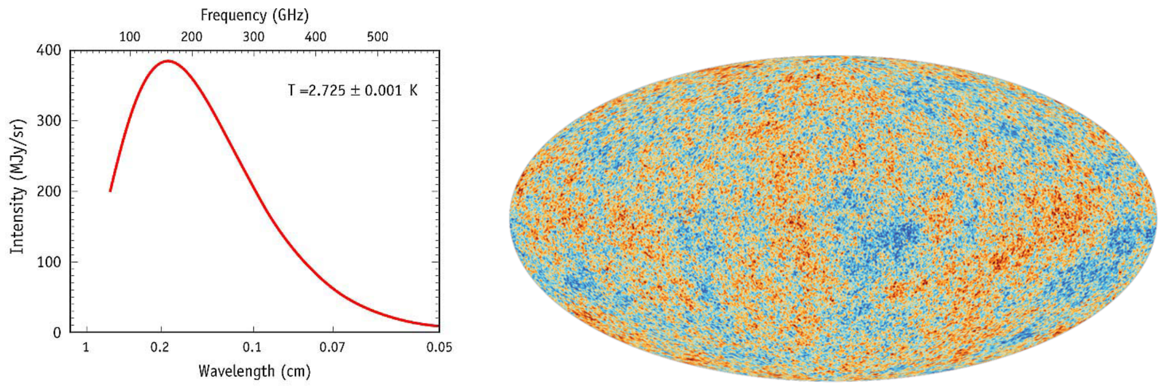


Figure 2: *Left*: The CMB has been measured to have an almost perfect black-body spectrum. Credit: NASA/WMAP Science Team. *Right*: The Planck satellite was launched to measure the tiny deviations from a perfect black-body. This is the map of the sky from their final data release from July 2018. Credit: ESA/Planck Collaboration.

The relative size of the Universe, called the scale factor a, varies with redshift as $a = a_0(1+z)^{-1}$, whilst in a matter dominated universe (a valid assumption after the CMB was emitted) the scale factor varies with time t as $a \propto t^{2/3}$.

(a) If the scale factor today is defined to be $a_0 \equiv 1$, calculate the age of the Universe when the CMB was emitted. [Current age of the Universe $t_0 = 13.8$ Gyr.]

[3]

(b) What was the temperature of the CMB when the Earth formed 4.5 Gyr ago?

[2]

Section C: Long Answer

Each long question is worth 10 marks. Answer **either** *Qu 13* **or** *Qu 14.* **Total: 10 marks**.

Iron Planet

13. Given the mass and radius of an exoplanet we can determine its likely composition, since the denser the material the smaller the planet will be for a given mass (see Figure 3). At a minimum orbital distance, called the Roche limit, tidal forces will cause a planet to break up, so very short period exoplanets place meaningful constraints on their composition as they must be very dense to survive in that orbit. The most extreme variant is called an 'iron planet', as it is made of iron with little or no rocky mantle ($\gtrsim 80\%$ iron by mass). The exoplanet KOI 1843.03 has the shortest known orbital period of a little over 4 hours, so is a potential candidate to be an iron planet.

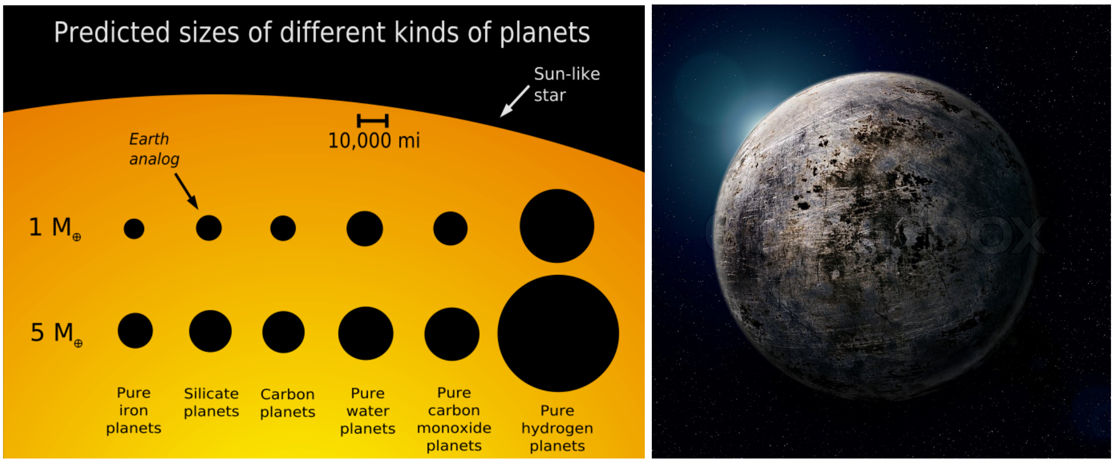


Figure 3: *Left*: Comparisons of sizes of planets with different compositions [Marc Kuchner / NASA GSFC]. *Right*: An artist's impression of what an extreme iron planet (almost $\sim 100\%$ iron) might look like.

The Roche limiting distance, a_{min}, for a body comprised of an incompressible fluid with negligible bulk tensile strength in a circular orbit about its parent star is

$$a_{\text{min}} = 2.44 R_\star \left(\frac{\rho_\star}{\rho_p}\right)^{1/3} ,$$

where $R_\star$ is the radius of the star, $\rho_\star$ is the density of the star and ρ_p is the density of the planet. Over the mass range $0.1\, M_E - 1.0\, M_E$ the mass-radius relation for pure silicate and pure iron planets is approximately a power law and can be described as

$$\log_{10}\left(\frac{R}{R_E}\right) = 0.295 \log_{10}\left(\frac{M}{M_E}\right) + \alpha ,$$

where $\alpha = 0.0286$ in the pure silicate case and $\alpha = -0.1090$ in the pure iron case.

(a) Derive a formula for the period of an exoplanet orbiting at the Roche limit, and hence show it is dependent only on the variable ρ_p (i.e. independent of the properties of the parent star). [2]

(b) The exoplanet KOI 1843.03 is measured (from transit lightcurves) to have a radius of $0.61\ R_E$. Calculate the minimum period for the exoplanet in the pure silicate and pure iron cases. Give your answer in hours. [6]

(c) The measured period is 0.1768913 days. If it is only made of iron and silicate, estimate the minimum percentage of iron, and hence conclude if it is an example of an iron planet. [2]

GPS and Relativity

14. Special Relativity (SR) tells us that two observers will disagree about the duration of a time interval measured by each one's clock if one is moving at speed v relative to the other, a phenomenon called time dilation. General Relativity (GR) tells us that gravitational fields dilate time too. This has an impact on satellites, since they travel at high orbital speeds (*slowing down* their clocks relative to the surface) but due to their altitude they are in a weaker gravitational field (*speeding up* their clocks relative to the surface). Which effect is dominant varies with orbital radius. Global Positioning System (GPS) satellites must compensate for this effect, since the satellites rely on accurate measurements of the time between sending and receiving a radio signal.

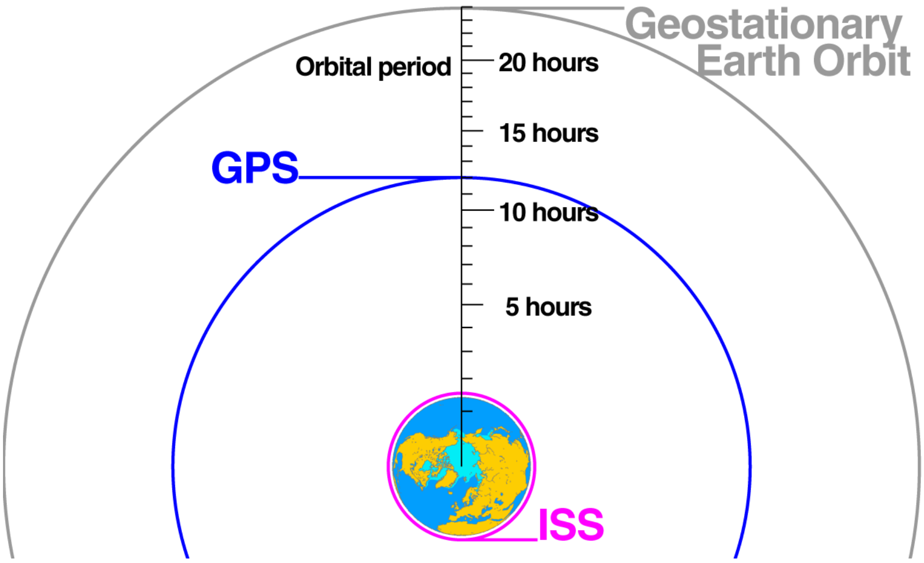


Figure 4: A scale diagram of the positions of the orbits for the International Space Station (ISS), GPS satellites and geostationary satellites, along with their orbital periods

In SR, time dilation can be calculated with

$$t' = \gamma t_0 \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{so} \quad \Delta t_{\text{SR}} = t_0 - t' = (1-\gamma)t_0 \ ,$$

where t_0 is the time measured by the moving clock, t' is the time measured by the observer, c is the speed of light and v is the speed of the object. A negative Δt indicates that the clocks are passing time slower relative to the observer, whilst a positive indicates they are passing quicker.

(a) GPS satellites have a period of exactly half a day. Use this to determine their orbital speed and hence show that for them $\Delta t_{\text{SR}} \approx -7$ μs when $t_0 = 1$ day.

[3]

In GR, the overall effect (taking into account both the orbital motion and changing gravitational field strength) can be calculated by considering the measurements of time passing on the surface of the Earth and on the satellite as taken by an observer infinitely far away from Earth (so outside the gravitational field). It can be shown that

$$\Delta t_{\text{overall}} = (\Gamma_{\text{GPS}} - \Gamma_{\text{E}})t_0 \quad \text{where} \quad \Gamma_{\text{GPS}} = \sqrt{1-\frac{3GM_E}{a_{\text{GPS}}c^2}} \quad \text{and} \quad \Gamma_{\text{E}} = \sqrt{1-\frac{2GM_E}{R_E c^2}} \ .$$

(b) Given a_{GPS} is the radius of the GPS satellite's orbit, calculate $\Delta t_{\text{overall}}$ when $t_0 = 1$ day. Give your answer in μs and state the significance of the sign.

[3]

(c) Hence calculate the daily offset in GPS positions if relativity was not taken into account.

[1]

(d) Sketch how $\Delta t_{\text{overall}}$ varies with orbital radius, r, from $r = R_E$ (near the ISS) to $r = 7\,R_E$ (near geostationary satellites), clearly indicating any important points.

[3]

[Hint: You may find the binomial approximation useful: $(1+x)^n \approx 1 + nx$ when $|x| \ll 1$]

END OF PAPER

Questions proposed by:
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