BAAO
British Astronomy and Astrophysics Olympiad

Astronomy & Astrophysics A2 Challenge

September – December 2016

Instructions

Time Allowed: **One hour**

In Section C, you can choose to answer **either** Q13 on the Saturn's rings, **or** Q14 on the discovery of the nearest exoplanet.

Marks allocated for each question are shown in brackets on the right.

You will need to use a ruler.

You may use any calculator.

You may use any standard formula sheet.

This is the first paper of the British Astronomy and Astrophysics Olympiad in the 2016/2017 academic year. To progress to the next stage of the BAAO, you must take BPhO Round 1 in November, which is a general physics problem paper.

To be awarded the highest grade (Distinction) in this paper, it should be sat under test conditions and marked papers achieving 60% or above should be sent in to the BPhO Office at Oxford by Wednesday 9th Nov 2016.

To solve some of the questions, you will need to write equations, draw diagrams and, in general, show your working. You are also encouraged to look at the clear sky and identify the brightest stars, a few days before sitting the paper.

This paper has more than an hour's worth of questions. You are encouraged to have a go at as many as you can and to follow up on those that you do not complete in the time allocated.

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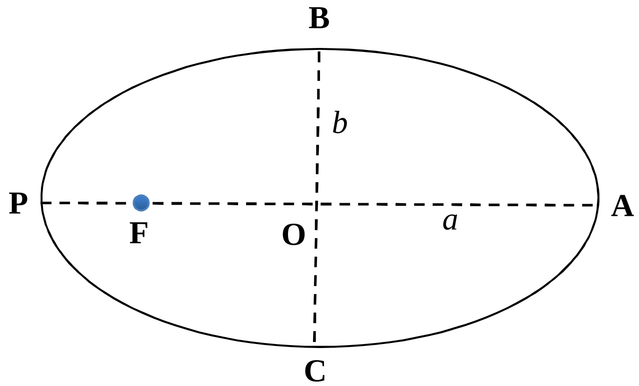
Useful constants

Speed of light	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	6.67×10^{-11}	$\mathrm{N\,m^{2}\,kg^{-2}}$
Solar mass	$M_{\odot}$	1.99×10^{30}	kg
Solar radius	$R_{\odot}$	6.95×10^{8}	m
Astronomical Unit	AU	1.496×10^{11}	m
Light year	ly	9.46×10^{15}	m
Earth's orbit semi-major axis		1	AU
Earth's orbital period	1 year	365.25	days
Earth's rotation period	1 day	24	hours
Earth's mass	M_{Earth}	5.97×10^{24}	kg
Earth's radius	R_{Earth}	6.37×10^{6}	m
Earth's axial tilt		23.4	°

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

a = OA (=PO) - semi-major axis
b = OB (=CO) - semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$ - eccentricity
F –- focus
P – periapsis (point nearest to F)
A – apoapsis (point furthest from F)

Kepler's Third Law:

For an elliptical orbit, the square of the period (T) of orbit of an object about the focus is proportional to the cube of the semi-major axis (a) (the average of the minimum and maximum distances from the Sun). The constant of proportionality is $4\pi^2/GM$, where M is the mass of the central object.

The questions were proposed by:

Dr Alex Calverley (Bedford School)
Sandor Kruk (University of Oxford)

Section A: Multiple Choice

*Write the correct answer to each question. Each question is worth 1 mark. There is only one correct answer to each question. Total: **10 marks**.*

1. Why do hurricanes rotate anti-clockwise in the northern hemisphere and clockwise in the southern hemisphere?

   A. Due to the Earth rotating from East to West
   B. Due to the different ratios of land to water area between the two hemispheres
   C. Due to the Moon's orbit being inclined by 5° above the ecliptic giving it more influence on the northern hemisphere
   D. Due to the Coriolis Effect causing paths of particles to curve as they travel over the Earth's surface

2. Why are there *two* tides every day?

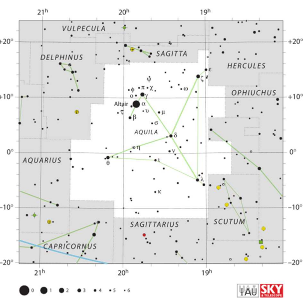
   A. The Moon causes the one at night and the Sun causes the one during the day
   B. The Earth and Moon orbit a common centre of mass
   C. The Moon is tidally locked to the Earth
   D. Water waves can only travel around the Earth in 12 hours

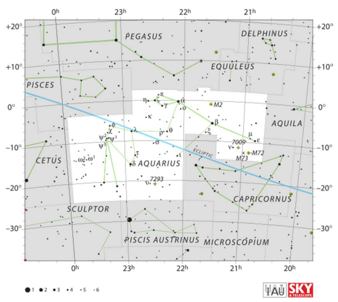
3. A typical cheap handheld telescope has a diameter of 10 cm, whilst ones for keen amateurs can have diameters of 40 cm. How much greater light gathering power does the larger telescope have?

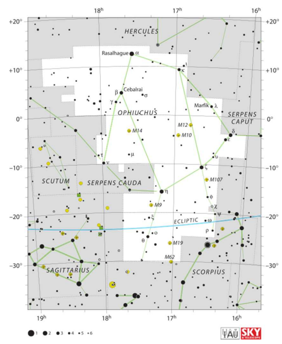
   A. 2
   B. 4
   C. 8
   D. 16

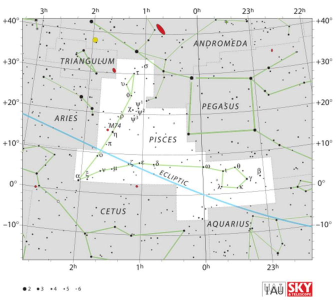
4. Which of the following is not a zodiacal constellation, according to the astronomical definition?

A. Aquila B. Aquarius C. Ophiucus D. Pisces









5. When observing from the UK, during which season is the Full Moon visible highest in the sky?

A. Spring
B. Summer
C. Autumn
D. Winter

6. Given an overwhelmingly large piece of paper, with a thickness of 10 μm, approximately how many times do you need to fold it in half (theoretically!) for the thickness of the final stack to reach from Earth to the Sun (1 AU)?

A. 40
B. 50
C. 60
D. 70

7. At a latitude of 52° N what is the altitude of Polaris above the horizon?

A. 38°
B. 48°
C. 52°
D. 90°

8. "Manhattanhenge" is the name given to when, just before sunset or just after sunrise 4 times a year (twice for setting, and twice for rising), the Sun aligns with the east-west streets of the New York grid system. One of the setting dates this year was on 11th July. Which of these is another date you are likely to see the "Manhattanhenge" sunset?

A. 30^{th} May
B. 30^{th} June
C. 11^{th} December
D. 11^{th} January

9. You travel 100 miles South, 100 miles East and 100 miles North and arrive back where you started. Where are you? You are NOT at the North Pole.

A. South Pole
B. 100 Miles from the North Pole
C. 116 Miles from the South Pole
D. 200 Miles from the South Pole

10. On 5^{th} September 2016, the Rosetta mission has finally found the Philae lander on Comet 67P/Churyumov–Gerasimenko. Considering that Philae (1×1×1 m) appeared in an image from the high-resolution camera (with 2048 × 2048 pixels and field of view 2.2° × 2.2°) as 25 × 25 pixels, from what distance did Rosetta manage to image Philae?

A. 1.1 km
B. 2.1 km
C. 12.2 km
D. 26.8 km

E. Section B: Short Answer

*Each short question is worth 5 marks. Total: **10 marks**.*

Question 11 Forces of Nature

In the BBC programme *Forces of Nature*, Brian Cox uses a Eurofighter Typhoon to try and overtake the spin of the Earth such that the setting Sun appears to rise instead.

a. By considering the circumference of the Earth at each point, and the length of a day, what speed in the air would the Eurofighter need to achieve for the Sun to appear stationary if it took off from:
   i. The equator
   ii. Oxford (which has a latitude of about 52°)
   [You can ignore the altitude of the fighter jet]

[2 marks]

b. A Eurofighter sets off from the equator just as the top edge of the Sun has gone below the horizon, and rapidly accelerates due west up to a speed of 500 m s^{-1}. Given that the Sun has an angular diameter of 0.5° as viewed from Earth, what is the minimum amount of time the fighter jet needs to fly for in order to see the whole of the Sun above the horizon?

[3 marks]

Question 12 Star Wars Rogue One

The new Star Wars film *Rogue One* concentrates on the creation of the first Death Star, which in one scene causes a total eclipse on the planet Scarif, where it is being built.

a. Assume the Death Star is being built in orbit around the Earth instead, but still causes a very brief total solar eclipse when it passes in front of the Sun. The Death Star has a diameter of 120 km, and so by comparing it to the size and distance to the Sun, calculate the altitude it is being built at. How does that compare to the altitude of the International Space Station (400 km)?

[3 marks]

b. What will be its orbital period, assuming it moves in a circular orbit?

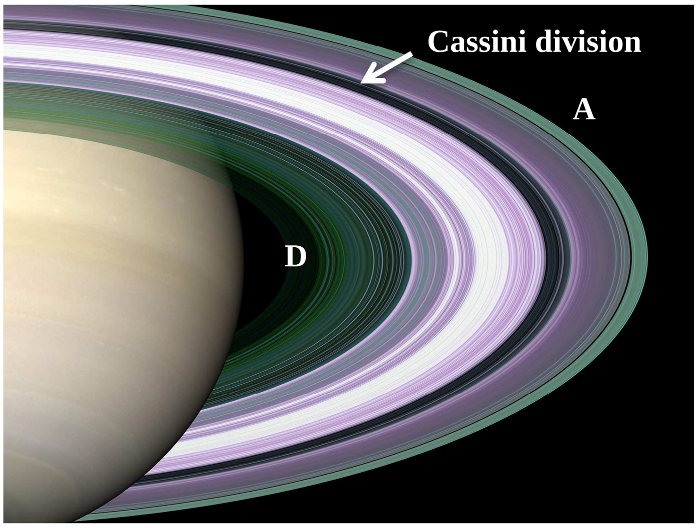
[2 marks]

Section C: Long Answer

*Each long question is worth 10 marks. Answer **either** Qu 13 **or** Qu 14. Total: **10 marks**.*

Question 13 Saturn's Rings

One possible theory for why the gas giants have ring systems is that a small moon got too close to the parent planet. When the gravitational tidal forces (due to the difference between the strength of the planet's pull on the near and far sides of the moon) became greater than the gravitational forces holding the moon together, it was ripped apart. This minimum distance is called the "Roche limit", named after the French astronomer Edouard Roche who first calculated it. It is defined as when the gravitational force generated by the moon at its surface is equal to the tidal forces it experiences at that distance.



Consider a spherical planet with mass M and radius R, and a perfectly rigid spherical moon with mass m and radius r, orbiting the planet in a circular orbit of radius d. For a small particle of mass u on the surface of the moon, the gravitational and tidal forces it experiences will be

$$F_{grav} = \frac{Gmu}{r^2} \qquad\qquad F_{tidal} = \frac{2GMur}{d^3}$$

a. By making these two expressions equal, derive an expression for the Roche limit, d_{RL}, purely in terms of R and the uniform densities of the planet and the moon (ρ_P and ρ_m respectively)

[4 marks]

b. Use your formula to calculate the Roche limit of Saturn for a moon made of water ice (ρ_m = 930 kg m^{-3}), given that M_{Saturn} = 5.68 × 10^{26} kg and R_{Saturn} = 60 270 km

[2 marks]

c. In practice, as a moon approaches the Roche limit it will start to deform and become more of an ellipsoid than a sphere, causing the tidal forces to increase, and so the Roche limit from our simple model is really a minimum radius. The opposite extreme would be to assume that both the planet and moon are made of a fluid, and so can deform without resistance (this works well when looking at things like stars in close binary systems). In that situation it can be shown that the equivalent formula for the Roche limit becomes

$$d_{RL} \approx 2.44R\left(\frac{\rho_M}{\rho_m}\right)^{\frac{1}{3}}$$

Work out this new maximum value for the Roche limit for water ice around Saturn.

[1 mark]

d. The inner edge of Saturn's rings (D ring) occurs at 1.11 R_{Saturn}, and the outer edge of the A ring (the last main visible ring) is at 2.27 R_{Saturn}. Do the rings of Saturn fall (roughly) between the two Roche limits calculated for the extreme cases of a perfectly rigid and a fluid moon made of water ice?

[1 mark]

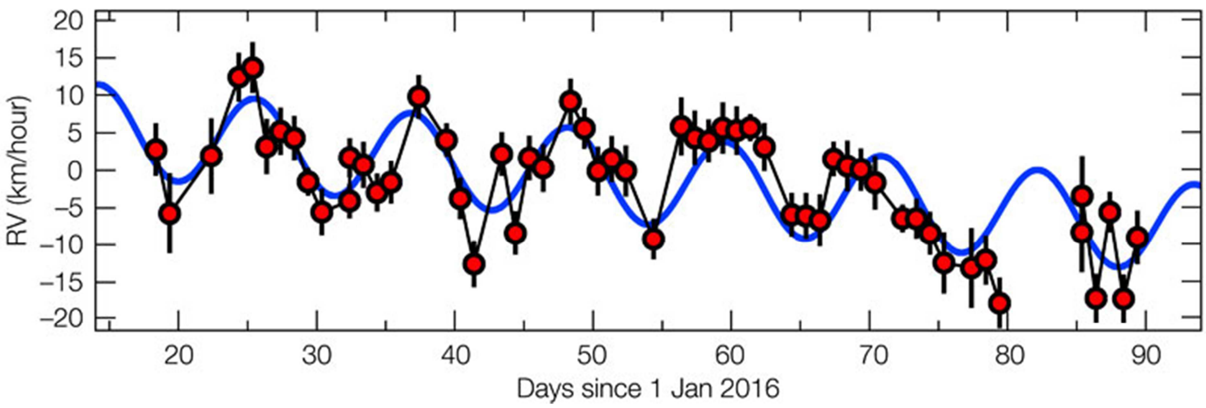
e. Edouard Roche was one of the first scientists to suggest the destruction of a moon (which he named Veritas) as a source of Saturn's rings. Assume that it used to orbit in what is now the Cassini Division (an apparent gap in the rings at around 2 R_{Saturn}), and that the fluid Roche limit was most relevant in this case. Given that the mass of the rings is 3.0 × 10^{19} kg, estimate the radius of Veritas.

[2 marks]

Question 14 Nearest exoplanet discovered

On 24th August 2016, astronomers discovered a planet orbiting the closest star to the Sun, Proxima Centauri, situated 4.22 light years away, which fulfils a long-standing dream of science-fiction writers: a world that is close enough for humans to send their first interstellar spacecraft.

Astronomers have noted how the motion of Proxima Centauri changed in the first months of 2016, with the star moving towards and away from the Earth, as seen in the figure below. Sometimes Proxima Centauri is approaching Earth at 5 km hour^{-1} – normal human walking pace - and at times receding at the same speed. This regular pattern of changing radial velocities caused by an unseen planet, which they named Proxima Centauri B, repeats and results in tiny Doppler shifts in the star's light, making the light appear slightly redder, then bluer.



a. From the radial velocity curve above, determine the period of the planet around Proxima Centauri.

[1 mark]

b. Proxima Centauri is a red dwarf star, unlike our Sun, with a mass of only 0.12 $M_{\odot}$. What is the semi-major axis of the planet's orbit in AU?

[1 mark]

c. Assuming that the orbit of Proxima Centauri B is circular, what is the planet's orbital velocity?

[1 mark]

d. By considering that the total linear momentum of the star-planet system in the centre of mass frame is zero, estimate the minimum mass of the planet in terms of Earth masses. Why is this a minimum for the mass of the planet?

[3 marks]

e. Using a simple approximation, the equilibrium temperature of a planet can be calculated as

$$T_{Planet} = T_{Star}\sqrt{\frac{R_{Star}}{2d}}$$

where d is the distance between the star and the planet. Given that the astronomers discovered that the orbit of the planet is in fact an ellipse with an eccentricity of 0.35, and that the star has a surface temperature of 3000 K and a radius of 0.14 $R_{\odot}$, what are the minimum and maximum equilibrium temperatures of Proxima Centauri B?

Comment on whether or not the planet is in the habitable zone of Proxima Centauri.

[The habitable zone is the band around a star where a planet can have water on its surface in liquid form, at normal pressure.]

[3 marks]

f. Comment on the prospects of studying the planet directly, during your lifetime, using robotic space probes.

[1 mark]

END OF PAPER