13th Singapore Astronomy Olympiad

Theory

14 March 2026

Instructions

1. The theory portion of this Olympiad is worth a total of **123 marks**.
2. When asked to do so, check that you have **10** printed pages.
3. Write your answers and workings clearly on the answer sheets provided.
4. Submit all used answer sheets.
5. Fill in these details on each side of your answer sheet:
   - Year of competition
   - Your participant code
   - The page number – which should be continuous from 1 to N
   - The question number
6. Cross out all workings or answers you do not wish to be evaluated.
7. If you require assistance (e.g. to visit the restroom, enquire about an ambiguity or possible errata, etc.), please get the attention of the invigilators.

Competition Rules and Regulations

1. Only the use of scientific calculators is permitted. No graphing or programmable calculators are allowed.
2. Disruptive behaviour, cheating, collusion to cheat or any integrity-related offences are grounds for immediate disqualification.
3. You may opt to retain the question paper and constants sheet for personal use. Return all unused answer sheets to the Organising Team.

The 13th Singapore Astronomy Olympiad is jointly organised by Astronomy.SG and the NTU Astronomical Society.

This page has been intentionally left blank

1 Stellar Aberration [20]

Stellar aberration occurs when the observed position of a star changes due to the motion of the observer. The details of some stars are listed in the table below. By choosing one (or more) star(s), determine the orbital velocity of Earth. Justify your choice of star(s). [20m]

Star	Annual variation of declination	Right Ascension (mean)	Declination (mean)
Eltanin (γ Dra)	$39''$	$17^h 57^m$	$51°29'$
Alkaid (η UMa)	$36''$	$13^h 48^m$	$49°11'$
Shedar (α Cas)	$34''$	$0^h 42^m$	$56°40'$
Mirfak (α Per)	$23''$	$3^h 26^m$	$49°57'$
Capella (α Aur)	$16''$	$5^h 18^m$	$46°01'$

Note: Use relativistic treatment even at small speeds as a varying speed of light is not physically correct.

2 Binary Accretion [48]

Consider a white dwarf of mass M_W travelling through a gas of uniform density ρ with relative velocity v. By considering the inelastic impact of gas on a narrow wake behind the white dwarf, determine for the rate of accretion $\dot{M}$ of the white dwarf:

(ai) assuming hypersonic flow ($v \gg v_s$), [12m]
(aii) a lower bound. [20m]

A newly discovered binary system of a red giant and a white dwarf has parameters listed below:

White dwarf mass, $M_W = 1.30 M_\odot$
White dwarf radius, $R_W = 2550$ km
Companion stellar wind mass flux, $\dot{M}_{loss} = 2.0 \times 10^{-6} M_\odot/\text{yr}$
Companion stellar wind speed, $v_{wind} = 30$ km/s
Companion mass, $M_c = 2.0 M_\odot$
Semi-major axis, $a = 2.0$ AU.

(b) Calculate the range of possible accretion rates of the white dwarf, in solar masses per Earth year. [6m]

For this white dwarf, the critical pressure of accreted gas at its surface to trigger runaway nuclear fusion is 1.35×10^{18} Pa.
(c) Calculate the range of possible periods of recurrence of the nova that would be produced. [6m]

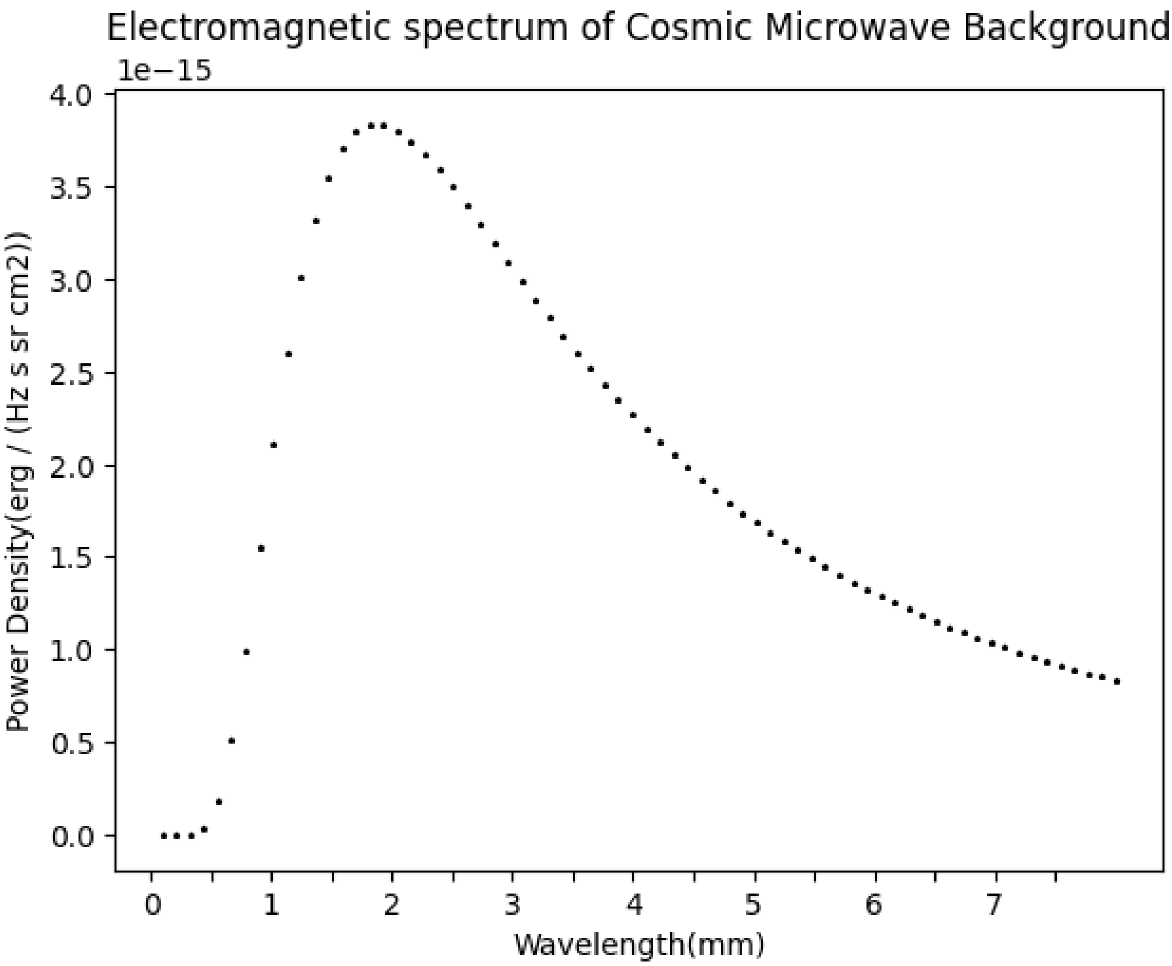
Explain qualitatively how changing the factors below would affect the recurrence period:
(di) Increase in white dwarf luminosity. [2m]
(dii) Increase in metallicity of accreted gas. [2m]

3 Back in my Days.. [12]

The cosmic microwave background radiation (CMB) is an electromagnetic signal measured in all directions, regardless of any light source present.

According to Big Bang Cosmology, the photons that make up the CMB are a relic of an early era in the universe, when it was in a hot, dense, opaque state, in constant equilibrium. The photons were released when the universe cooled to about 3000 Kelvin and was transparent to photons, known as the recombination era. These photons traveled untouched as the universe expanded and cooled before reaching us in its current state.

The spectrum of the CMB as measured now is shown below.



(a) Given this, how much has the universe expanded since the CMB photons were released? [2m]

(b) During the time of recombination, the universe can be treated as a homogenous, isotropic fluid that is in thermal and energetic equilibrium. What was the energy density of radiation in the universe then? [5m]

Hint: Consider how all angles of light in a blackbody radiate through a chosen flat surface within a blackbody to fill its volume.

According to the ΛCDM model of the universe, the expansion of the universe is linked to its energy density, which is contributed by matter, radiation and dark energy, with dark energy having a constant energy density.

(c) Given that the current energy density of matter and dark energy is $2.82 \times 10^{-10} J/m^3$ and $6.00 \times 10^{-10} J/m^3$ respectively, what was the energy density of these 2 components during the era of recombination? [3m]

(d) Making the wild assumption that the universe's expansion is constant, what would have been the age of the universe during Recombination? [2m]

4 Brighter than the Sun [19]

In the year 2050, astronomers discovered a binary star system consisting of a Wolf-Rayet Star and a white dwarf. Wolf-Rayet stars are extremely active stars that have strong stellar winds and violent outbursts which occur at random.

Stellar fusion in the Wolf-Rayet Star is primarily carried out via the CNO cycle. The CNO cycle process is shown as follows:

$${}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma$$

$${}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + e^+ + \nu_e$$

$${}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{14}_{7}\mathrm{N} + \gamma$$

$${}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{15}_{8}\mathrm{O} + \gamma$$

$${}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + e^+ + \nu_e$$

$${}^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He}$$

The table below lists the data on the mass of certain isotopes of different elements involved in the CNO Cycle.

Isotope	Mass
${}^{1}_{1}\mathrm{H}$	1.007825031 amu
${}^{4}_{2}\mathrm{He}$	4.002603254 amu
${}^{12}_{6}\mathrm{C}$	12.0000000 amu
${}^{13}_{6}\mathrm{C}$	13.0033548 amu
${}^{15}_{7}\mathrm{N}$	15.0001089 amu

(ai.) Given that the WR Star produces 5.61×10^{24} kg of helium every year, calculate its maximum luminosity. [5m]

(aii.) Briefly explain why its actual luminosity is likely lower than what that was calculated in (ai.). [2m]

(b) This star has a radius of $12R_\odot$. Calculate its peak wavelength. [3m]

For this star, it loses approximately $0.0003M_\odot$ to the white dwarf per year, of which 35% is due to accretion from the ejected stellar winds. The remainder was accreted directly from the stellar envelope of the WR star.

For simplicity, assume that the density of a shell of the WR star is inversely proportional to the square of its distance from the center of the star. We will also assume that the WR star is approximately at its Eddington limit [formula provided below].

$$L_{\mathrm{Edd}} \approx 5.5 \times 10^4 \left(\frac{M}{M_\odot}\right) L_\odot$$

Every 64 years, a nova is observed from this system. The mass of the white dwarf right before the nova was determined to be $1.28M_{\odot}$. The white dwarf releases just enough material to return back to its initial mass after each nova.

The radius of a white dwarf can be related to its mass by the following equation:

$$\text{WD} \approx 7.8 \times 10^6 \text{ m} \times \left[\left(\frac{1.4\,\text{M}_{\odot}}{M_{\text{WD}}} \right)^{2/3} - \left(\frac{M_{\text{WD}}}{1.4\,\text{M}_{\odot}} \right)^{2/3} \right]^{1/2}$$

(c) Find the orbital radius of the white dwarf. Assume a circular orbit and ignore the initial velocity of the particles accreted by the white dwarf. [9m]

Hint: This formula calculates the distance of the Roche Lobe R_L, where a is the semi major axis.

$$R_L = a\,\frac{0.49\,q^{2/3}}{0.6\,q^{2/3} + \ln(1 + q^{1/3})}, \quad q = \frac{M_{\text{WD}}}{M_{\text{WR}}}$$

5 Space Shenanigans [24]

JWST was launched on December 25 2021 from Kourou, French Guiana 5°N.

(a) Assuming that earth is a perfect sphere of radius 6.370×10^6 m, find the tangential velocity at the launch site due earth's rotation. [2m]

To plan out the trajectory of JWST to its final position around Lagrange Point 2, Scientist A wanted to make a rough calculation using Satellite Bob, a simulated model.

Satellite Bob is in orbit around the earth.

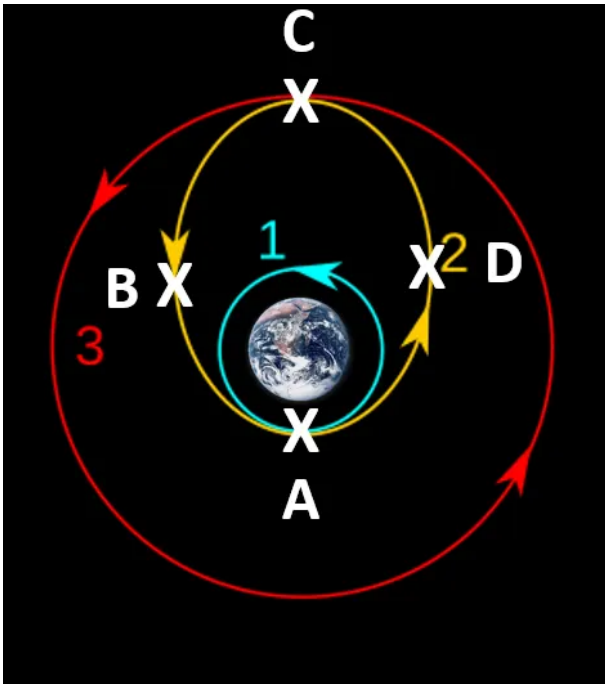


Figure 1: Satellite Bob's transfer orbit

(b) There are two burns conducted in order for a Hohmann transfer for Satellite Bob, which starts at the innermost orbit, orbit 1. Write in order, the burns conducted. eg. E, F (as marked by the crosses and letters A–D in the image). [2m]

(c) Satellite Bob's orbit 1 is 2000 km from the surface of the Earth and orbit 3 is 40 000 km from the surface of earth, find the Δv made during the first and second Hohmann transfers. List your answer in the format (Δv of transfer 1, Δv of transfer 2). Assume that both orbits are circular and the $M(\text{earth}) = 5.972 \times 10^{24}$. [7m]

(d) Calculate the total energy of Satellite Bob at Orbit 3, given the Mass(satellite) is 6500 kg. [3m]

(e) JWST is set to orbit around Lagrange point 2 (L2). Find an equation describing the balance of forces at L2. Express your ans in terms of G, $M_{\odot}$, M_{Earth}, R, and x. [4m]

One of JWST's tasks is to observe exoplanets, to learn more about their atmospheric composition. One example is the WASP-39 system. Here, we'll discuss the study of WASP-39b, one of the planets in this system. The following is a transmission spectrum of the transit of WASP-39b. (only for visual reference, don't use numbers from the image to answer the following questions)

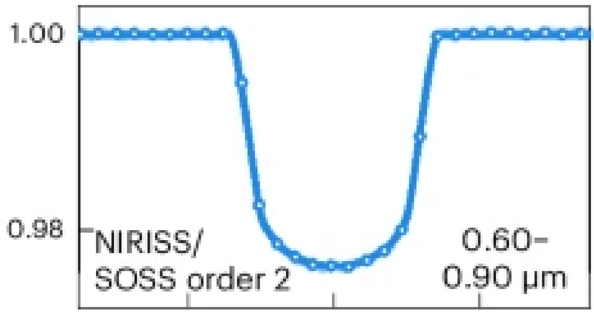


Figure 2: Transmission spectrum of WASP-39b transit - relative flux against time. Only for visual reference, don't use numerical values in the image to answer.

The following parameters will be useful for solving parts f and g.

Mass of WASP-39	0.93 $M_\odot$
Mass of WASP-39b	0.28 $M_{Jupiter}$
Radius of WASP-39b	1.27 $R_{Jupiter}$
Transit distance	$0.047AU$
Peak wavelength of WASP-39	$537nm$
Orbital inclination of WASP-39b (as viewed by JWST)	87.83°
Distance from earth to WASP-39	$700ly$
Ratio of the transit distance of WASP-39b to the diameter of WASP-39	0.92388

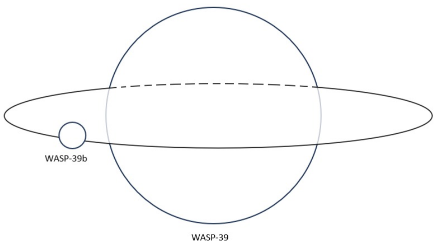


Figure 3: Transit distance

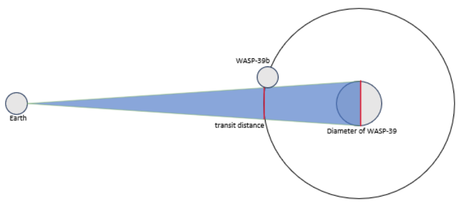


Figure 4: Ratio of the transit distance of WASP-39b to the diameter of WASP-39

(f) Assume the transit lasted 120 min from first entry of the WASP-39b to the complete exit. Find the radius of the star WASP-39. [3m]

(g) Find the flux received by WASP-39b from WASP-39. [3m]