Data Analysis solutions

by Fish and Kane

Path of the Sun [25m]

(a)

Sunrise 7am [1m]
Sunset 7pm [1m]

Marker's comments:
Generally well done.
It was expected for contestants to simply read from the graph
Quite a few did not though their values were correct so marks were awarded in full as well.

(bi)

1m for each line representing each solstice and equinox
1m for correct labels

Marker's comments:
Generally well done.
Marks were lost for the lack of labelling but otherwise most contestants were able to correctly sketch the lines
Equinoxes being the same line was accepted so long as labels were given for both.

(bii)

2m for analemma definition
[1m for stating same **mean solar time** and location;
1m for stating the gist of it being a "figure 8 path of the sun traced across a year"]

0.5m for correct 12pm analemma label
0.5m for correct analemma drawn

Marker's comments:
Not as well done.
Many contestants did not state mean SOLAR time
Some also lacked labels for the analemma
0.5m were deducted for those that drew on Diagram B instead of A
Participants that drew overexaggerated analemmas on Diagram A were also not awarded the 0.5m for the analemma.

(biii)

0.5m for labels
0.5m for figure 8 shape
0.5m for having the N-side bigger than/the same as the S-side
0.5m for drawing it on the correct location on the graph

Marker's comments:
Reasonably well done.
Most mark deductions came from the 0.5m that was deducted for those that drew on Diagram B instead of A

(ci)

0.5m for each solstice/equinox 0.5m for each correct label

Marker's comments:
Not well done.
Many did not draw the solstices/equinoxes as curves but only as straight lines. Those that did draw curves had instances of drawing intersecting solstice/equinox lines which was rejected. Many also did not catch the location properly as New Zealand is South and therefore the Winter Solstice would in fact be above not below.

(cii)

1m for each analemma
[0.5m awarded for correct labels;
0.5m awarded for correct analemma]

Marker's comments:
Not well done.
Many skipped or could not identify the correct location or orientation of the analemma [since South the N loop of the analemma should be smaller than the S loop]

(di)

0.5m for each solstice/equinox 0.5m for each correct label

Marker's comments:
Not well done.
Many did not draw the solstices/equinoxes as curves but only as straight lines. Those that did draw curves had instances of drawing intersecting solstice/equinox lines which was rejected.

(dii)

1m for each analemma
[0.5m awarded for correct labels;
0.5m awarded for correct analemma]

Marker's comments:
Not well done.
Many skipped or could not identify the correct location of the analemma

Far Far Away [20m]

2.1 Candlelight

(a) Relatively straightforward, first find mean apparent magnitude of RR lyrae via number crunching to be $m = 15.611625$.

Then we use $m - M = 5\log\left(\frac{d}{10\text{pc}}\right)$, rearrange to get:

$$\begin{aligned} d &= (10\text{ pc}) \times 10^{\frac{m-M}{5}} \\ &= (10\text{ pc}) \times 10^{\frac{15.611625-0.75}{5}} \\ &= 9382.63884812623\text{ pc} \\ &= 30606.16792258776\text{ ly} \end{aligned}$$

1m was awarded for finding the correct apparent magnitude
1m for stating the correct equation
0.5m for correct signs

1m for answer
0.5m for units

Marker's comments:
Well done.
Except for the few that did not give in the correct units, most scored very well for this part of the question.

2.2 Bundles of stars

(b) Contestants are to circle where RR Lyrae stars are in the HR diagram. With the apparent magnitude given [around +15.6],and the knowledge that RR Lyrae are in the *Horizontal Branch* along the *Instability Strip*, it leaves little room to miss. However it is not really possible to tell accurately where the Instability Strip is, so **full 1 mark was awarded if they circled somewhere on the 2 patches on the horizontal branch around B-V of +0.0 to 0.5**.

Marker's comments:
Not well done.
Quite a few circled the MS turn off point while others drew unreasonably vague/large circles.
Both were rejected.
Others correctly identified the usage of the +15.6 apparent magnitude but got the B-V index wrong and therefore it was rejected as well.

(c) ***If signs or initial values are wrong give ECF and award remaining marks***

I'd accept Main Sequence (MS) turnoff apparent magnitude of M5 anywhere from $m = +17.5$ to $+18.0$. Accepted apparent magnitude of MS turnoff for M3 would be $m = +19 \pm 0.25$. Up to your discretion beyond these ranges – there's no grid after all. **Award 1 mark for each correct apparent magnitude.**

We now have the suspicious claim that M5 and M3 are the same age and as such their MS turnoff stars are of the same age and same absolute magnitude.

Contestants can go about finding $M_{\text{MS turnoff}}$ from M3 and then $m - M = 5\log\left(\frac{d}{10\text{pc}}\right)$ for the distance to M5.

$$\begin{aligned} M_{\text{MS turnoff}} &= m - 5\log\left(\frac{d_{\text{M3}}}{10\ \text{pc}}\right) \\ &= 19 + 5\log\left(\frac{9382.63884812623}{10}\right) \\ &= +4.138375 \end{aligned}$$

1 mark for correct working, and 1 mark for correct result.

Marker's comments:
Not the most well done.
Many participants did not understand the question which led to the use of other methods such as TRGB or taking MS turn off and proceeding with only that.
Some also failed to identify the need to find MS turn off entirely.
For those that used TRGB, partial marks were awarded.

(a) ***If signs or initial values are wrong give ECF and award remaining marks***

$$\begin{aligned} d_{\mathrm{M5}} &= (10\ \mathrm{pc}) \times 10^{\frac{m-M}{5}} \\ &= 4702.458808416159\ \mathrm{pc} \\ &= 15.339420633053509\ \mathrm{kly} \end{aligned}$$

Contestants with elite ball knowledge can also go:

$$m_{\mathrm{M3}} - m_{\mathrm{M5}} = 5 \log\left(\frac{d_{\mathrm{M5}}}{d_{\mathrm{M3}}}\right)$$

which should yield the same result. Contestants with even more elite ball knowledge can do whatever as long as it is conceptually and mathematically sound.

1m was awarded for stating the initial equation
0.5m for stating the final equation
1m for correct signs
1m for the correct answer
0.5m for units

Marker's comments:
Reasonably well done though some contestants messed up the sign convention or failed to identify the correct values to be used

(e) $v = H_0 d$. **If correct working and answer (w/ ecf), award full 1 mark**

$$\begin{aligned} v &= H_0 d \\ &= (67.80\ \mathrm{km\ s^{-1} Mpc^{-1}}) \times \frac{721\ \mathrm{Mly}}{3.262 \frac{\mathrm{ly}}{\mathrm{pc}}} \\ &= 14985.836909871245\ \mathrm{km/s} \end{aligned}$$

(f) $z = \frac{v}{c} = 0.04998737129628273$

If correct working and answer (w/ ecf), award full 1 mark

Accept relativistic formulas, like $1 + z = \sqrt{\frac{c+v}{c-v}}$, or $z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} - 1 = 0.05130165614810189$

2.3 Turning Red

(g) $\frac{\lambda - \lambda_{\mathrm{rest}}}{\lambda_{\mathrm{rest}}} = z$.

ECF wil be given if z value in 5f was wrong

1m for correct shift % as reflected in graph
1m awarded for correct shift direction
1m for presence of the previously non-visible lines [see chart]
1m for drawing more than 3 lines correctly
1m for correctly drawing all lines

Marker's comments:
Overall reasonably well done.
We had cases of blueshifting instead of redshifting, though the most commonly occurring problem was that must did not realise that you are also required to sketch the lines of the previously invisible wavelengths that were shown in the chart
Some also did not draw all lines correctly despite having done the correct calculations, to that BOD was given and marks were still awarded