T1

1. (a) (1 point) We know $m_t - m_e = -2.5log\left(\frac{F_t}{F_e}\right)$

   (1 point) since,$F_{lim} \propto \dfrac{1}{D^2}$ so $m_t - m_e = 5log\,(1000/7)$

   $m_t = 10.8 + 6$

   (1 point) $m_t = 16.8$

   Now we know $m = M + 5log\left(\frac{r}{10pc}\right)$

   $\frac{r}{10} = 10^{3.454}$

   (2 points) So $r = 29Kpc$ is the maximum distance till which student can detect the star using telescope.

   (b) (2 points) $m' = m + A$,thus $m' = 16.8 + \alpha r$

   (1 point) Now,$m' = M + 5log\left(\frac{r}{10pc}\right)$ $16.8 + 0.00005 * r = -0.5 + 5log(r) - 5$

   (2 points) $r = 10^{4.46-0.00001r}$
   This equation can't be solve analytically.

   (2 points) So,if we do the iteration for this, taking initial value from previous result of 29Kpc, which is good approximation.

   (3 points) Then, we get r=18.73KPc.

T2

2. (2 points) $r_p = a(1-e)$ is the perihelion distance. $r_a = a(1+e)$ is the aphelion distance. Use the law of conservation of energy to get:

   (2 points)

$$E = -\frac{GMm}{r_p} + \frac{mv_p^2}{2} = -\frac{GMm}{r_a} + \frac{mv_a^2}{2}$$

   (1 point) From the conservation of angular momentum,

$$mv_a r_a = mv_p r_p$$

$$\implies v_a = v_p \frac{r_p}{r_a}$$

(3 points) Substitute velocities and rearrange the first equation to get (alternative expression making v_p the subject is also possible and correct):

$$v_a^2 \left(1 - \frac{r_a^2}{r_p^2}\right) = 2GM \left(\frac{1}{r_a} - \frac{1}{r_p}\right)$$

(3 points) Then, substitute this into the equation for the total mechanical energy:

$$E = \frac{-GMm}{r_a} + \frac{GMmr_p^2}{r_p^2 - r_a^2} \left(\frac{r_p - r_a}{r_a r_p}\right)$$

Which gives (after simplifying or any equivalent formulation):

$$E = \frac{-GMm}{2a} \left(\frac{2}{1+e} - \frac{1-e}{1+e}\right)$$

(2 points) Finally giving us:

$$E = -\frac{GMm}{2a}$$

(2 points) Substitute r_a and r_p in the energy equations to get:

$$\frac{v_a}{v_p} = \frac{1-e}{1+e}$$

(or use the conservation of angular momentum directly).

T3

3. (a) (3 points)

$$lon_g = \frac{5.75}{12} \times 180 = 86.25 \deg E$$

(4 points)

$$lon_k = lon_g - \frac{105 \times \cos\phi}{6378} \times \frac{180}{\pi} = 85.415 \deg E$$

(b) (3 points)

$$t_{diff} = \frac{105 \times \cos\phi}{6378} \times \frac{12 \times 3600}{\pi} \text{ s } = 200.4 \text{ s } = 3 \text{ min } 20.4\text{s}$$

$$t_{ktm} = 8{:}35 \text{ pm} - t_{diff} = 8{:}31{:}40$$

(1 point) Answer can be written as 8:32pm, or 8:31:40pm.

(c) (2 points) Ravi has the same local time as Kathmandu. On his watch, however, his time zone is UTC +5:30. Thus, the difference will be 15 - t_{diff} mins, which is 11 mins 40 (39.6) seconds.

(2 points) As per his watch, therefore, the occultation will occur at 8:16:40 pm, or 8:17 pm.

T4

4. (a) (2 points) $L \propto P^{2.5} \rightarrow L = Kp^{2.5}$, taking log on both side then $\log L = \log K + 2.5 \log P$

$\log L = \log F + \log 4 + \log \pi + 2 \log r$

(1 points) $m_1 - m_2 = -2.5 log \left(\frac{F_1}{F_2} \right)$

For ease of calculation we can use Vega as reference star and m be apparent magnitude of cepheid variable then

(1 point) $\frac{-m}{2.5} = \log F_1 - \log F_o$

$m = M + 5 log \left(\frac{r}{10pc} \right)$

Solving these equation we get,

(3 points)

$$M = -6.25 LogP + C$$

,where $C = -2.5(LogK - Log4 - LogF_o - Log\pi) + 5Log10$

(b) (1 point) So for above equation of magnitude luminosity relation C=-1.5,

(1 point) we suppose, m is the average magnitude for luminosity, $\frac{1}{2}(L_{\max} + L_{\min})$.

(1 point) Now $m - m_{ref} = -2.5 \log \left(\frac{\frac{1}{2}(L_1 + L_2)}{L_{ref}} \right)$

$m - m_{ref} = 2.5 \log 2 - 2.5 \log \left(\frac{L_1 + L_2}{L_{ref}} \right)$

$m = m_{ref} + 2.5 \log 2 - 2.5 \log \left(\frac{L_1 + L_2}{L_{ref}} \right)$

(1 point) $\frac{L_{tot}}{L_{ref}} = \left(\frac{L_1+L_2}{L_{ref}}\right) = \left(\frac{L_1}{L_{ref}} + \frac{L_2}{L_{ref}}\right)$

$$= \left(10^{-0.4m_1} + 10^{-0.4m_2}\right) 10^{0.4m_{ref}}$$

(1 point) Now substituting the values,
$m = 2.5 \cdot \log 2 - 2.5 \cdot \log(10^{-\frac{2}{5}m_1} + 10^{-\frac{2}{5}m_2})$

(1 point) From graph $m_{max} = 28$ and $m_{min} = 22$ thus m=22.75 and period is 20 days.

(1 point) So, using the Absolute magnitude relation with peiod we obtained above we can write M=-9.63.

(1 point) Substitute the value in this equation $m = M + 5log\left(\frac{r}{10pc}\right)$ we get, r=29.92 Mpc.

T5

(3 points) Firstly, calculate the geocentric orbit height by using Newton's law of gravitation and setting T = 23 hours 56 mins 4 secs.

$$\frac{GMm}{d^2} = \frac{mv^2}{d} \text{ and } v = \frac{2\pi d}{T}$$

(1 point) Solve for d to get:

$$d = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$

(1 point) Substitute the correct values to obtain $d = 4.216 \times 10^7$m.

(5 points) Then, construct a triangle, such that from the law of sines we have:

$$\frac{\sin(90+a)}{d} = \frac{\sin(90-(\phi+a))}{r}$$

where r is the radius of the Earth, a is the altitude and ϕ is the latitude of Kathmandu.

(2 points) Simplifying:

$$\cos(a)\frac{r}{d} = \cos(\phi)\cos(a) - \sin(\phi)\sin(a) \implies 1 - \tan(\phi)\tan(a) = \frac{r}{d\cos(\phi)}$$

(2 points) Rearranging and solving for a, you get:

$$a = \tan^{-1}\left(\frac{1 - \dfrac{r}{d\cos(\phi)}}{\tan(\phi)}\right)$$

(1 point)Plugging in the values:

$$a = 57.64\circ$$

T6

1. (1 point) The mean synodic period of the Moon is 29.53059 days. Dashain happened 12 synodic periods earlier, so

   (3 points) $12 \times 29.53059 = 354.36708$ days $= 354$ days 8 hours 48 minutes.

   (1 point) Because 2024 is a leap year, there are 366 days between the two 12 Octobers.

   (3 points) Dashain in 2023 will be 354 days before the 12th of October.

   (2 point) In other words, it was 12 days after the 12 October, 2023, which was 24 October, 2023.

2. (1 point) Dashain in 2025 will be 354 days 8 hours 48 minutes after 11am on 12 October 2024,

   (2 points) which is 11 days before 12 October 2025 at around 7:48pm on 1 October 2025.

   (2 points) 4 days and 12 hours after this is 7:48am on 6 October 2025. Therefore, *Kojagrat Purnima* will fall on 6 October 2025.

T7

1. (1 point)

   $$\lambda_{obs} = (1+z) \times \lambda_{rest} = 3.379 \times 656.3\text{nm}$$

   (1 point)

   $$= 2217.6\text{nm}$$

2. (2 points) Lensing is multiplicative on the flux. So:

   $$m_{corr} = m_{obs} + 2.5\ \log(30) = 24.19$$

3. (2 points)
$$f_{obs} = 3631 \times 10^{-\frac{20.5}{2.5}} \mathrm{Jy} = 2.29 \times 10^{-5} \mathrm{Jy}$$
(2 points)
$$f_{corr} = \frac{f_{obs}}{\mu} = 7.64 \times 10^{-7} \mathrm{Jy}$$

4. (2 points) $\Sigma_{UV} = f_{obs}/13$.

   (2 points) In magnitudes, $\Sigma'_{UV} = -2.5\log(\Sigma_{UV}/3631) = 23.3$ mag arcsec^{-2}

5. (3 points)
$$r = \sqrt{13 \times 8^2/\pi\mu} = 3 \text{ kpc}$$
[1 point for correct placement of magnification factor, 1 for squaring scale, 1 for correct answer.]

T8

1. (3 points) We know:
$$\sin(a) = \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(H)$$
(1 point) At rise, a = 0.
$$\cos(H) = -\tan(\phi)\tan(\delta)$$
(3 points) Solve for H to get $H = 5$ hours 49 minutes 59 seconds.
(2 points)
$$t_{rise} = 12\text{pm } - H = 6{:}10 \text{ am}$$

2. (1 point) Since the stars are very close, you can use planar geometry to solve this problem.
$$d = \sqrt{(\Delta\alpha)^2 + (\Delta\delta)^2}$$
(2 point) Substitute $\Delta\alpha = 0.08376$ rad
(2 points) $\Delta\delta = 0.01927$ rad
(1 points) Solving in the original equation, obtain $d = 0.08595$ rad or in degrees, 4.92 deg .
Answers that round to 4.9 degrees are acceptable. Answers in degrees or subunits there of for any part are acceptable. Correct solutions with spherical trigonometry will also receive full points.

T9

1. (2 points) Notice that the expression for the acceleration due to gravity becomes:
$$a = -\frac{4}{3}G\pi\rho r$$

which can be expressed as:

$$a = -k^2 r, \text{ where: } k = \sqrt{\frac{4}{3} G\pi\rho}$$

(1 point) This is the equation for a harmonic oscillator, as given in the hint. The time period is given by:

$$T = \frac{2\pi}{k}$$

(2 points) which simplifies to:

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

2. (3 points) From the expression of V, the potential difference between r = R and r = 0 is given by $\Delta V_{int} = \dfrac{GM}{2R}$

   (2 points) Finally, to escape:

$$\frac{1}{2} m v_{esc}^2 - \frac{4}{3} G m \rho \pi R^2 - \Delta V_{int} = 0$$

   (5 points) Collecting terms and simplifying in terms of v_{esc}:

$$v_{esc} = 2R\sqrt{G\pi\rho}$$

T10

1. (1 point) We know $V_1 = \omega a_1$ and $V_2 = \omega a_2$.

   (2 points) From Kepler's Third law: $P^2 = \frac{4\pi^2}{GM} a^3$, where $M = m_1 + m_2$ and $a = a_1 + a_2$.

   (1 point) We know that $P = \frac{2\pi}{\omega}$, $a_1 = \frac{m_2}{(m_1+m_2)} a$ and $a_2 = \frac{m_1}{(m_1+m_2)} a$

   (2 points) Replacing above value in Kepler's equation we get,

   $G(m_1 + m_2) = 4\pi^2 \frac{\omega^2}{4\pi^2} \frac{(m_1+m_2)^3}{m_2^3} a_1^3$

   $\frac{m_2^3}{(m_1+m_2)^2} = (\omega a_1)^3 \frac{P}{2\pi G}$

   (2 points) When we observe the star we can't measure the real orbital velocity but the component of it as the plane of orbit and astronomer line of sight have angle θ,thus

(1 point) $\frac{m_2^3}{(m_1+m_2)^2}(sin^3\theta) = (v_{1obs})^3\frac{P}{2\pi G}$, which is required mass function, where v_{1obs} is maximum observed orbital velocity

(1 point) similarly we can write another mass function, $\frac{m_1^3}{(m_1+m_2)^2}(\sin^3\theta) = (v_{2obs})^3\frac{P}{2\pi G}$

2. (1 point) We know $\frac{v_1}{v_2} = \frac{m_2}{m_1}$

   (2 points) From graph we can see that $\frac{v_1}{v_2} = 1.5$ thus $m_2 = 1.5m_1$, period is 40 days and v_{1obs} is 75 km/s,and for minimum value we consider $\sin\theta$ as 1,if we put these value in mass function, we get

   (1 point) $0.54m_1 = 1.74M_{\odot}$

   $m_1 = 3.2M_{\odot}$,similarly

   (1 point) $m_2 = 4.9M_{\odot}$

T11

1. (1 point) Use the escape velocity definition:

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

   (1 point) together with Hubble's law:

$$v = H_0R$$

   And:

$$M = \frac{4}{3}\pi R^3\rho_c$$

   (4 points) Substitute Hubble's Law and Mass in the escape velocity definition to obtain:

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

2. (2 units)

$$H_0 = 2.2685 \times 10^{-18}\ \mathrm{s}^{-1}$$

   (2 points) Get: $\rho_c = 9.20 \times 10^{-27}\mathrm{kg\ m}^{-3}$

3. (1 point)

$$M = \frac{4}{3}\pi\rho_cR_{\odot}^3$$

(1 point)

$$N = \frac{M}{M_{tennis}}$$

(3 points) Substitute values to get:

$$N = 89.45 \text{(either 89 or 90}$$

(both acceptable).

T12

1. (2 points)

$$\lambda_{max} T = 2.898 \times 10^{-3} mK$$

(1 point) Thus, $\lambda_{max} = 57.96 \mu m$.

2. (2 points) Use the Stefan-Boltzmann Law and compare to the Sun to get:

$$R_{dust} = R_{\odot} \sqrt{\frac{L_{dust}}{L_{\odot}} \frac{T_{\odot}^4}{T_{dust}^4}}$$

(2 points) Substituting values, you get:

$$R_{dust} = 120.4 \text{pc}$$

3. (3 points)

$$d_{dust} = \frac{cz}{H_0} = \frac{0.01 \times 2.998 \times 10^5}{70} Mpc = 42.83 \text{ Mpc}$$

(2 points)

$$\theta = \frac{R_{dust}}{d_{dust}} rad = 0.580\text{”}$$

4. (3 points)

$$D > 1.22 \frac{\lambda'_{max}}{\theta} = 25.4 \text{m}$$

λ_{max} needs to have redshift accounted, i.e. $\lambda'_{max} = (1 + z)\lambda_{max}$ (or some equivalent). Only 1 point gained if λ_{max} is used for correct formula.