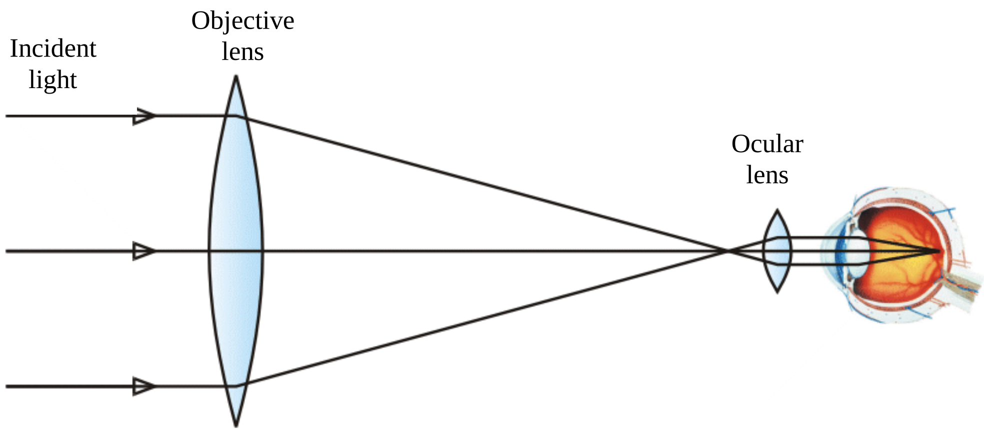
Grid 1. The stars observed with the telescope

The diameter of the objective lens of a telescope, represented in the drawing in figure 1, is $D_{\text{objective telescope}} = 300$ mm, and the diameter of the pupil of the observer's eye is $D_{\text{pupileye}} = 6$ mm.

The degree of collection of light, coming from a star, due to this telescope, is:



a) $g_c = 2500$; b) $g_c = 1500$; c) $g_c = 1000$; d) $g_c = 3000$.

Solution

The degree of collection of light from a star, g_c, by a telescop, used as shown in the drawing in Figure 1, is defined as the ratio of the area of the circle's surface, whose diameter is equal to the diameter of the lens of the telescope, and the area of the circle, whose diameter is equal to the diameter of the pupil of the observer's eye:

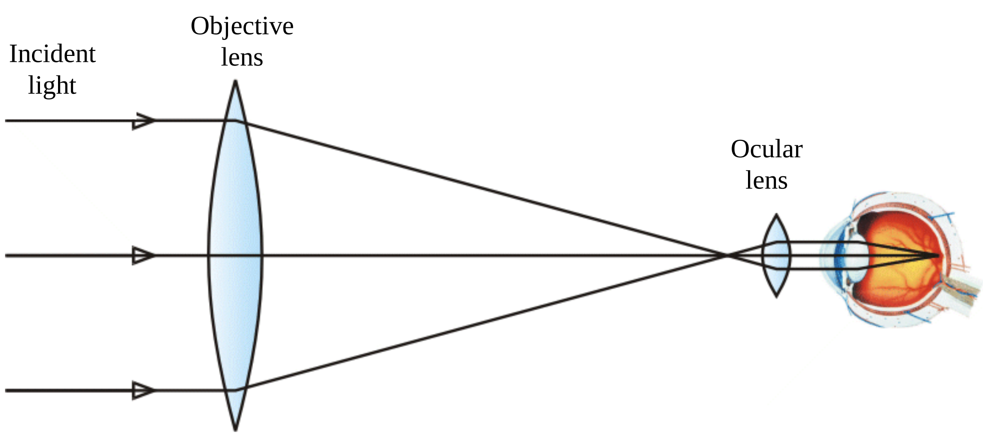


Fig. 1

$$g_c = \frac{\frac{\pi}{4} D^2_{\text{objectivtelescope}}}{\frac{\pi}{4} D^2_{\text{pupileye}}} = \left(\frac{D_{\text{objectivtelescope}}}{D_{\text{pupileye}}} \right)^2 ;\ D_{\text{objectivtelescope}} > D_{\text{pupileye}};\ g_c > 1.$$

Grid 2. The Sun on the Horizon

The astronomical refraction correction, ρ_{r}, has the minimum value, $\rho_{\mathrm{r,min}}=0$, when the star is at the zenith of the Earth's surface observer $\left(z=0; h=90^{0}\right)$, and the maximum value of the astronomical refraction correction, $\rho_{\mathrm{r,max}}$, is made when the star, whose light passes through the Earth's atmosphere, to reach the Earth's surface observer. $\left(z=90^{0}; h=0\right)$, is on the horizon when the star rises or sets

It is known that, for an observer, atmospheric refraction raises the Sun in the moments of sunrise and sunset, that is, when it is below the horizon, at its limit, approximately, with an apparent disk**.**

Knowing : the radius of the Sun, $R_{\mathrm{S}}=6{,}96\cdot10^{5}$ km; the distance between the center of the Earth and the center of the Sun, $d=15\cdot10^{7}$ km, then :

a) $\rho_{\mathrm{r,max}}=23{,}7'$; b) $\rho_{\mathrm{r,max}}=43{,}7'$;

c) $\rho_{\mathrm{r,max}}=53{,}7'$; **d)** $\rho_{\mathrm{r,max}}=33{,}7'$.

Solution

Atmospheric refraction raises the Sun above its horizon, at its limit, at sunrise and sunset, approximately, with an apparent disk, as shown in the drawing in Figure 1.

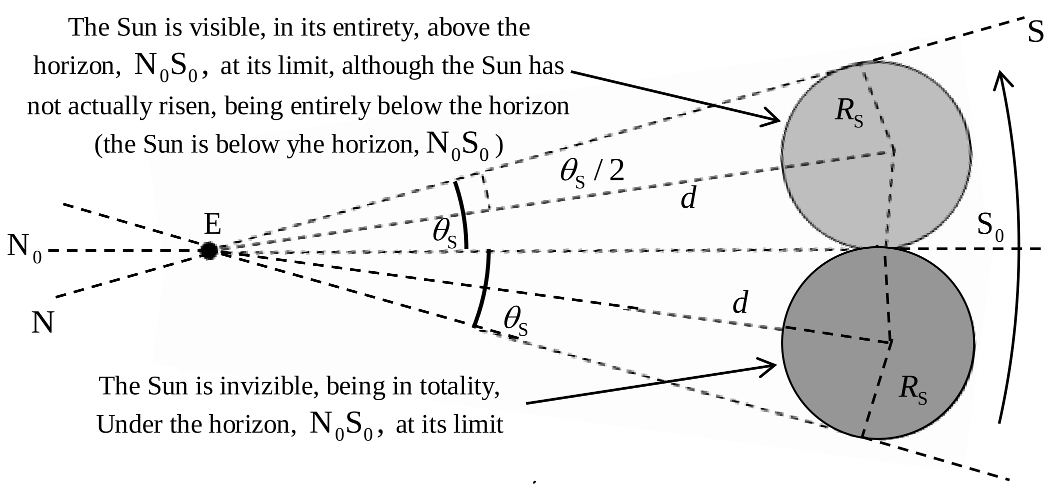


Fig. 1

Therefore, in reality, the rising of the upper arc of the Sun's disk begins after the moment when we already see the entire solar disk above the horizon. We see the sunrise and sunset, later than they actually occur, which makes the day a little longer.

I mean, I see the disk of the whole Sun above the horizon, at its limit, but the Sun has not yet risen, it is still below the horizon !

I mean, the sun went down, it went down below the horizon, at the edge of it, but I still see the whole disk of it above the horizon !

Under these conditions, the astronomical refraction has the maximum value :

$$\rho_{\mathrm{r,max}} = \theta_{\mathrm{S}};$$

$$\sin\frac{\theta_{\mathrm{S}}}{2} = \frac{R_{\mathrm{S}}}{d} \approx \frac{\theta_{\mathrm{S}}}{2};$$

$$\theta_{\mathrm{S}} = \frac{2R_{\mathrm{S}}}{d} = \frac{2\cdot 6{,}96\cdot 10^{5}\ \mathrm{km}}{15\cdot 10^{7}\ \mathrm{km}} = 0{,}0098\ \mathrm{radiani};$$

$$1\,\mathrm{rad} = \frac{180\cdot 60'}{3{,}14};$$

$$\theta_{\mathrm{S}} = 0{,}0098\cdot\frac{180\cdot 60'}{3{,}14} \approx 33{,}7' = \rho_{\mathrm{r,max}},$$

which proves that the value of the maximum astronomical refraction correction, $\rho_{\mathrm{r,max}}$, is approximately equal to the apparent angular diameter of the Sun, θ_{S}.

Grid 3. The third cosmic speed

Given: $V_0 \approx 30\dfrac{\text{km}}{\text{s}}$, the speed of the Earth in its circular orbit around the Sun; $v_0 \approx 7{,}9\dfrac{\text{km}}{\text{s}}$, the speed of a terrestrial satellite orbiting the Earth in a very low circular orbit (the first cosmic speed).

It is known that: $\dfrac{M_{\text{Earth}}}{R_{\text{Earth}}} << \dfrac{M_{\text{Sun}}}{R_{\text{Earth-Sun}}}$.

The variation of the kinetic energy of the body, in relation to the Sun, is neglected during the evolution of the body from the surface of the Earth to the limit of the gravitational attraction of the Earth.

The approximate minimum escape velocity that must be imparted to a body, B, relative to the Earth, launched from the Earth so that it leaves the Solar System forever (the third cosmic velocity), is:

a) $v_B \approx 45{,}52\dfrac{\text{km}}{\text{s}}$; b) $v_B \approx 32{,}32\dfrac{\text{km}}{\text{s}}$;

c) $v_B \approx 22{,}22\dfrac{\text{km}}{\text{s}}$; **d)** $v_B \approx 42{,}42\dfrac{\text{km}}{\text{s}}$.

Solution

Let $\vec{v}_B$ the speed of body B at the time of its launch from Earth, in relation to the Sun, so that the body reaches the limit of the gravitational attraction of the Sun and there it is at rest in relation to the Sun. Using the details in Figure 1, in accordance with the law of conservation of mechanical energy, it follows:

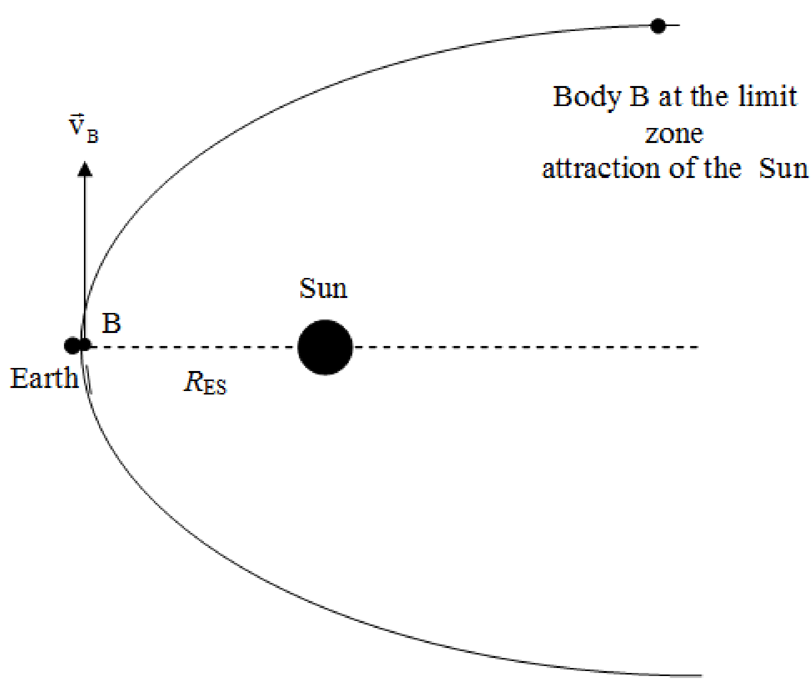


Fig. 1

$$R_E << R_{ES};$$

$$\frac{m\mathrm{v}_{\mathrm{B}}^{2}}{2}-K\frac{mM_{\mathrm{E}}}{R_{\mathrm{E}}}-K\frac{mM_{\mathrm{S}}}{R_{\mathrm{ES}}}=0;$$

$$\frac{M_{\mathrm{E}}}{R_{\mathrm{E}}}<<\frac{M_{\mathrm{S}}}{R_{\mathrm{ES}}};$$

$$\frac{m\mathrm{v}_{\mathrm{B}}^{2}}{2}-K\frac{mM_{\mathrm{S}}}{R_{\mathrm{ES}}}=0;$$

$$\mathrm{v}_{\mathrm{B}}=\sqrt{2}\sqrt{K\frac{M_{\mathrm{S}}}{R_{\mathrm{ES}}}};$$

$$\sqrt{K\frac{M_{\mathrm{S}}}{R_{\mathrm{ES}}}}=\mathrm{v}_{\mathrm{ES}}=V_{\mathrm{orbital}}=V_{0},$$

representing the orbital speed of the Earth on the circular trajectory around the Sun;

$$\mathrm{v}_{\mathrm{B}}=\sqrt{2}\,V_{0},$$

representing the second cosmic speed in relation to the Sun (parabolic speed);

$$R_{\mathrm{ES}}\approx 1.5\cdot 10^{8}\ \mathrm{km};\ T_{\mathrm{ES}}=1\,\mathrm{year};$$

$$V_{0}=\frac{2\pi R_{\mathrm{ES}}}{T_{\mathrm{S}}}\approx 30\,\frac{\mathrm{km}}{\mathrm{s}};\ \mathrm{v}_{\mathrm{B}}\approx 42.42\,\frac{\mathrm{km}}{\mathrm{s}}.$$

Conclusion: the body launched from the Earth, reaches the limit of the gravitational attraction of the Sun, in relation to the Sun, on a parabolic trajectory, with the Sun in its focus.

Grid 4. The Terrestrial Satellite

A small satellite (a luminous sphere) that orbits the Earth, in an equatorial circular orbit at the altitude h, is photographed with a camera on the ground, the lens of which is a converging lens with focal length f. The photography is made when the satellite aligns with the camera and the center of the Earth.

Given: the mass of the Earth, M; the constant of gravitational attraction, K. During the photographic exposure the proper rotation of the Earth is neglected.

The length of the image on the photograph, after developing the photographic plate, placed in the focal plane of the objective lens, as indicated by the drawing in figure 1, if the exposure time of the camera was Δt, knowing that the radius of the Earth is R, is given by the expression:

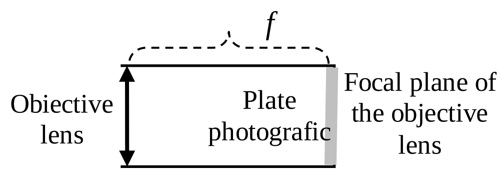


Fig. 1

a) $l = \frac{f}{h-f} \cdot \sqrt{K\frac{M}{R+h}} \cdot \Delta t$; b) $l = \frac{f}{h+f} \cdot \sqrt{K\frac{M}{R+h}} \cdot \Delta t$;

c) $l = \frac{f}{h-f} \cdot \sqrt{K\frac{M}{h-R}} \cdot \Delta t$; d) $l = \frac{f}{h-f} \cdot \sqrt{2K\frac{M}{R+h}} \cdot \Delta t$.

Solution

Sector AB, in the circular orbit of the satellite, represented in the drawing in Figure 2, traveled by the satellite during the time Δt, equal to the exposure time of the camera, has the length:

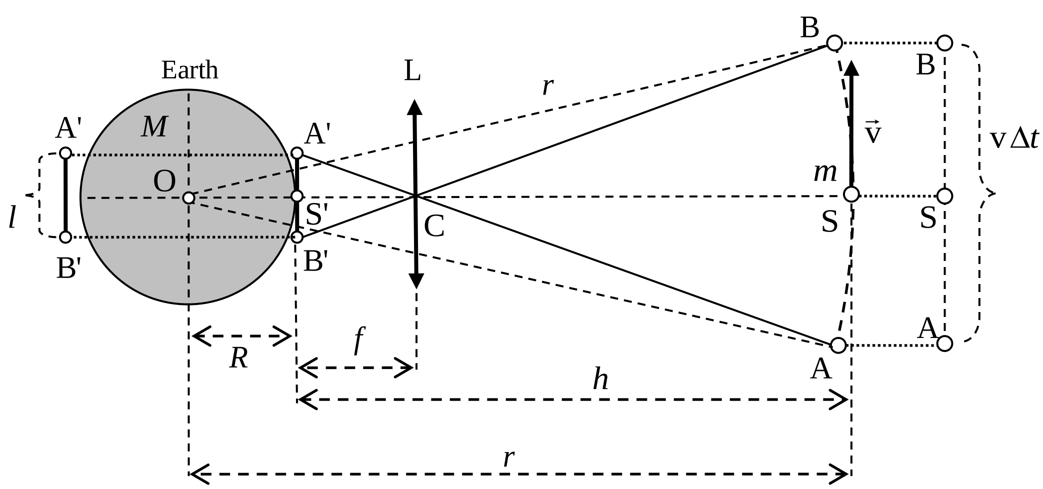


Fig. 1

$$AB = v_{rel} \cdot \Delta t,$$

where $\text{v} -$ the speed of the satellite in its circular orbit, relative to the center of the Earth;

$$K\frac{mM}{r^2} = \frac{m\text{v}^2}{r};$$

$$\text{v} = \sqrt{K\frac{M}{r}} = \sqrt{K\frac{M}{R+h}}.$$

Because the distance between the satellite and the camera lens is very large, the image will be formed in the focal plane of the camera lens.

Since the circle sector AB is very short and has a very large radius (r = R + h), it can be assimilated with the line segment AB, so that:

$$\Delta(\text{ACS}) \sim \Delta(\text{A'CS'});$$

$$\frac{\text{A'S'}}{\text{AS}} = \frac{\text{CS'}}{\text{CS}}; \frac{\frac{l}{2}}{\frac{\text{v}\cdot\Delta t}{2}} = \frac{f}{h-f},$$

where $l -$ the length of the image on the photo;

$$l = \frac{f}{h-f}\cdot\text{v}\cdot\Delta t;$$

$$l = \frac{f}{h-f}\cdot\sqrt{K\frac{M}{R+h}}\cdot\Delta t.$$

Grid 5. Andromeda Constellation Rotation

Our galaxy and the constellation Andromeda can be observed with the help of a telescope whose mirror has a diameter of $D = 6$ m.

The following are known: the Earth - Andromeda distance, $R = 1.42 \cdot 10^{11} R_0$, where R_0 is the radius of the Earth's circular orbit around the Sun; the mass of our Galaxy, $M_{\mathrm{G}} = 2.5 \cdot 10^{11} M_0$, where M_0 is the mass of the Sun; $T_0 = 1$ terrestrial year, the period of rotation of the Earth around the Sun; the mass of the constellation Andromeda, $M_{\mathrm{A}} = 3.6 \cdot 10^{11} M_0$.

Knowing that:

1) the photos were taken in visible light, with a wavelength $\lambda = 5 \cdot 10^{-7}$ m;

2) the angular distance between two objects for which they can be observed separately is $\varphi_0 = 1.22 \cdot \dfrac{\lambda}{D}$.

Using the photographs thus obtained, time required to highlight the rotational motions of our galaxy and the constellation Andromeda around their common center of mass, is:

a) 10^4 ani; **b)** 10^3 ani; c) 10^5 ani; d) 10^4 ani.

Solution

From the study of light diffraction it is known that the angular distance between two objects for which they can be observed separately is approximately. λ / D. It means that the change in the position of the constellation Andromeda can be observed if its angular displacement is:

$$\varphi_0 = 1.22 \cdot \frac{\lambda}{D} = 1.22 \cdot \frac{5 \cdot 10^{-7}}{6} = 10.16 \cdot 10^{-8} \text{radians.}$$

If T is the period of rotation of our Galaxy and the Andromeda Constellation, respectively, around their common center of mass, then the time required for the angular displacement φ_0 of the Andromeda constellation is:

$$\tau = \frac{\varphi_0}{\omega} = \frac{\varphi_0}{2\pi} T.$$

For two binary systems, using the generalized form of Kepler's third law, we can write that:

$$\left(\frac{T_1}{T_2}\right)^2 \left(\frac{M_1 + m_1}{M_2 + m_2}\right) = \left(\frac{a_1}{a_2}\right)^3,$$

out of which, for the galaxy - Andromeda and Sun - Earth systems, results in:

$$\left(\frac{T}{T_0}\right)^2 \left(\frac{M_{\mathrm{G}} + M_{\mathrm{A}}}{M_0 + M_{\mathrm{P}}}\right) = \left(\frac{R}{R_0}\right)^3; \; M_{\mathrm{P}} << M_0; \, T_0 = 1 \text{ terrestrial year;}$$

$$\left(\frac{T}{T_0}\right)^2 \left(\frac{M_{\mathrm{G}} + M_{\mathrm{A}}}{M_0}\right) = \left(\frac{R}{R_0}\right)^3;$$

$$T^2 = T_0^2 \cdot \left(\frac{R}{R_0}\right)^3 \cdot \left(\frac{M_0}{M_G + M_A}\right); \; \tau = \frac{\varphi_0}{2\pi} T; \; \varphi_0 = 1.22 \cdot \frac{\lambda}{D};$$

$$\tau = 1.22 \cdot \frac{\lambda}{2\pi D} T_0 \left(\frac{R}{R_0}\right)^{3/2} \left(\frac{M_0}{M_{\mathrm{G}} + M_{\mathrm{A}}}\right)^{1/2} \approx 10^3 \text{ years.}$$

Grid 6. The Sun as seen from Saturn

It is known that: the distance Saturn - Sun is 9.54 times greater than the distance Earth - Sun; the angular diameter of the Sun's disk, seen from Earth, is 32'.

The angular diameter of the Sun seen from Saturn, is:

a) $\alpha_{Sun-Saturn} \approx 0.001$ radiani; b) $\alpha_{Sun-Saturn} \approx 0.002$ radiani;
c) $\alpha_{Sun-Saturn} \approx 0.003$ radiani; d) $\alpha_{Sun-Saturn} \approx 0.004$ radiani.

Solution

Since the distance between Saturn and the Sun is 9.54 times greater than the distance from Earth to the Sun, it turns out that the angular diameter of the Sun's disk, observed from Saturn, is 9.54 times smaller than the angular diameter of the Sun, observed from Earth. so that, using the drawing in Figure 1, it results:

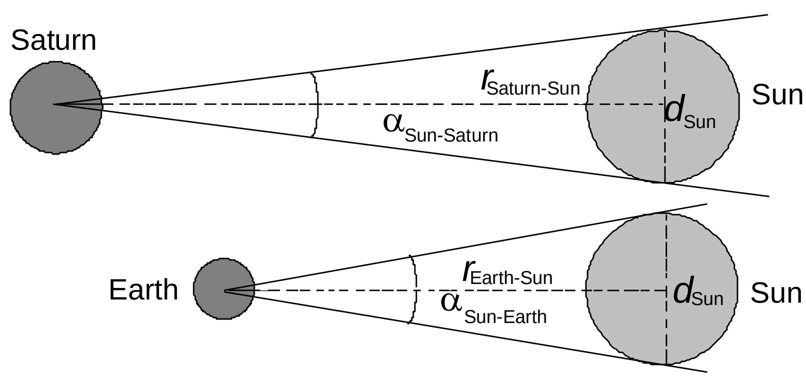


Fig. 1

representing the angular diameter of the Sun seen from Saturn.

Grid 7. Balls Suspended inside a Terrestrial Satellite

An artificial satellite evolves around the Earth in a circular orbit with radius r, permanently maintaining the same orientation towards the Earth, as shown in the drawing in figure 1. Inside the satellite are suspended, by very light wires, four identical spherical balls, each with mass m, such that the balls (b) and (c) are symmetrical to the ball (a), the balls (a) and (d) are symetrical to the ball (c), and the difference of their distances to the center of the Earth is $\Delta r = 2d$.

Given: $M -$ the mass of the Earth; $K -$ gravitational attraction constant.

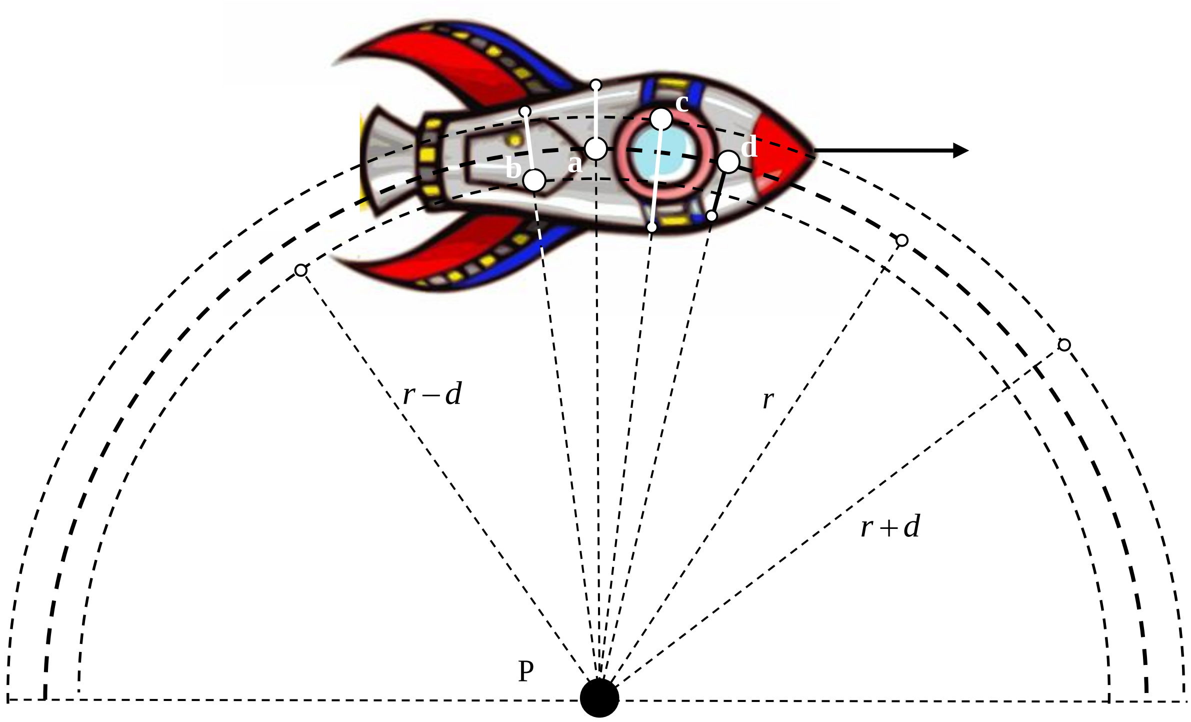


Fig. 1

The tension in each suspension wire, is:

a) $T_{(\mathrm{a})} = 0$; $T_{(\mathrm{b})} = 3K\dfrac{mMd}{r^3} > 0$; $T_{(\mathrm{c})} = 3K\dfrac{mMd}{r^3} > 0$; $T_{(\mathrm{d})} = 0$;

b) $T_{(\mathrm{a})} = K\dfrac{mMd}{r^3}$; $T_{(\mathrm{b})} = K\dfrac{mMd}{r^3} > 0$; $T_{(\mathrm{c})} = K\dfrac{mMd}{r^3} > 0$; $T_{(\mathrm{d})} = K\dfrac{mMd}{r^3}$;

c) $T_{(\mathrm{a})} = 3K\dfrac{mMd}{r^3} > 0$; $T_{(\mathrm{b})} = 0$; $T_{(\mathrm{c})} = 0$; $T_{(\mathrm{d})} = 3K\dfrac{mMd}{r^3} > 0$;

d) $T_{(\mathrm{a})} = 2K\dfrac{mMd}{r^3} > 0$; $T_{(\mathrm{b})} = 2K\dfrac{mMd}{r^3} > 0$; $T_{(\mathrm{c})} = 2K\dfrac{mMd}{r^3} > 0$; $T_{(\mathrm{d})} = 2K\dfrac{mMd}{r^3} > 0$.

Solution

The orientation of the spacecraft being constant, and it is, as a whole, a rigid solid body, means that all points belonging to the spacecraft move around the Earth at the same angular velocity, ω, whose expression is obtained as follows:

$$F_{\mathrm{ag(NC)}} = K\frac{m_{\mathrm{(NC)}}M}{r^2} = m_{\mathrm{(NC)}}\omega^2 r;\;\; \omega^2 = K\frac{M}{r^3}.$$

The centripetal accelerations of the three suspended balls are:

$$a_{\mathrm{cp(a)}} = \omega^2 r;\;\; a_{\mathrm{cp(b)}} = \omega^2(r-d);\;\; a_{\mathrm{cp(c)}} = \omega^2(r+d);\;\; a_{\mathrm{cp(d)}} = \omega^2 r = a_{\mathrm{cp(a)}}.$$

Assuming that all the suspension wires are tensioned, then, for each ball, the resultant of the gravitational pull force with the tension in the respective suspension wire is the centripetal force responsible for the circular motion of each ball.

Results that:

- f*or the ball* (a):

$$\vec{F}_{\mathrm{cp(a)}} = \vec{F}_{\mathrm{ag(a)}} + \vec{T}_{\mathrm{(a)}};\;\; F_{\mathrm{cp(a)}} = F_{\mathrm{ag(a)}} - T_{\mathrm{(a)}};\;\; T_{\mathrm{(a)}} = F_{\mathrm{ag(a)}} - F_{\mathrm{cp(a)}};$$

$$F_{\mathrm{ag(a)}} = K\frac{mM}{r^2};\; F_{\mathrm{cp(a)}} = ma_{\mathrm{cp(a)}} = m\omega^2 r;\; \omega^2 = K\frac{M}{r^3};\; F_{\mathrm{cp(a)}} = ma_{\mathrm{cp(a)}} = K\frac{mM}{r^2};$$

$$T_{\mathrm{(a)}} = K\frac{mM}{r^2} - K\frac{mM}{r^2};$$

$$T_{\mathrm{(a)}} = 0,$$

which means that the suspension wire of the ball (a) is not tensioned, so that the ball (a) is in a weightless state;

- *for the ball* (b):

$$\vec{F}_{\mathrm{cp(b)}} = \vec{F}_{\mathrm{ag(b)}} + \vec{T}_{\mathrm{(b)}};\;\; F_{\mathrm{cp(b)}} = F_{\mathrm{ag(b)}} - T_{\mathrm{(b)}};\;\; T_{\mathrm{(b)}} = F_{\mathrm{ag(b)}} - F_{\mathrm{cp(b)}};$$

$$F_{\mathrm{ag(b)}} = K\frac{mM}{(r-d)^2} = K\frac{mM}{r^2\left(1-\frac{d}{r}\right)^2} = K\frac{M}{r^2}\left(1-\frac{d}{r}\right)^{-2};$$

$$d << r;\;\; \left(1-\frac{d}{r}\right)^{-2} \approx 1 + 2\cdot\frac{d}{r};$$

$$F_{\mathrm{ag(b)}} = K\frac{M}{r^2}\left(1+2\cdot\frac{d}{r}\right);$$

$$F_{\mathrm{cp(b)}} = ma_{\mathrm{cp(b)}};\;\; a_{\mathrm{cp(b)}} = \omega^2(r-d);\;\; F_{\mathrm{cp(b)}} = m\omega^2(r-d);$$

$$\omega^2 = K\frac{M}{r^3};\; F_{\mathrm{cp(b)}} = K\frac{mM}{r^3}(r-d);$$

$$T_{\mathrm{(b)}} = K\frac{M}{r^2}\left(1+2\cdot\frac{d}{r}\right) - K\frac{mM}{r^3}(r-d) = K\frac{mM}{r^2} + 2K\frac{mMd}{r^3} - K\frac{mM}{r^2} + K\frac{mMd}{r^3};$$

$$T_{\mathrm{(b)}} = 3K\frac{mMd}{r^3} \neq 0;\;\; T_{\mathrm{(b)}} = 3K\frac{mMd}{r^3} > 0,$$

which means that the ball suspension wire (b) is tensioned so that the ball (b) is not weightless;

- *for the ball* (c)

$$\vec{F}_{\mathrm{cp(c)}} = \vec{F}_{\mathrm{ag(c)}} + \vec{T}_{\mathrm{(c)}};\;\; F_{\mathrm{cp(c)}} = F_{\mathrm{ag(c)}} + T_{\mathrm{(c)}};\;\; T_{\mathrm{(c)}} = F_{\mathrm{cp(c)}} - F_{\mathrm{ag(c)}};$$

$$F_{\mathrm{ag(c)}} = K\frac{mM}{(r+d)^2} = K\frac{mM}{r^2\left(1+\frac{d}{r}\right)^2} = K\frac{mM}{r^2}\left(1+\frac{d}{r}\right)^{-2};$$

$$d << r;\;\; \left(1+\frac{d}{r}\right)^{-2} \approx 1 - 2\cdot\frac{d}{r};$$

$$F_{\mathrm{ag(c)}} = K\frac{mM}{r^2}\left(1-2\cdot\frac{d}{r}\right);$$

$$F_{\mathrm{cp(c)}} = ma_{\mathrm{cp(c)}};\;\; a_{\mathrm{cp(c)}} = \omega^2(r+d);\;\; F_{\mathrm{cp(c)}} = m\omega^2(r+d);$$

$$\omega^2 = K\frac{M}{r^3};\; F_{\mathrm{cp(c)}} = K\frac{mM}{r^3}(r+d);$$

$$T_{\mathrm{(c)}} = K\frac{mM}{r^3}(r+d) - K\frac{mM}{r^2}\left(1-2\cdot\frac{d}{r}\right) = K\frac{mM}{r^2} + K\frac{mMd}{r^3} - K\frac{mM}{r^2} + 2K\frac{mMd}{r^3};$$

$$T_{\mathrm{(c)}} = -3K\frac{mMd}{r^3} \neq 0;\;\; T_{\mathrm{(c)}} = 3K\frac{mMd}{r^3} > 0,$$

which means that the ball suspension wire (c) is tensioned so that the ball (c) is not weightless;

- *for the ball* (d)

$$\vec{F}_{\mathrm{cp(d)}} = \vec{F}_{\mathrm{ag(d)}} + \vec{T}_{\mathrm{(d)}};\;\; F_{\mathrm{cp(d)}} = F_{\mathrm{ag(d)}} + T_{\mathrm{(d)}};\;\; T_{\mathrm{(d)}} = F_{\mathrm{cp(d)}} - F_{\mathrm{ag(d)}};$$

$$F_{\mathrm{ag(d)}} = K\frac{mM}{r^2};\; F_{\mathrm{cp(d)}} = ma_{\mathrm{cp(d)}} = m\omega^2 r;\; \omega^2 = K\frac{M}{r^3};\; F_{\mathrm{cp(d)}} = ma_{\mathrm{cp(d)}} = K\frac{mM}{r^2};$$

$$T_{\mathrm{(d)}} = K\frac{mM}{r^2} - K\frac{mM}{r^2};$$

$$T_{\mathrm{(d)}} = 0,$$

which means that the suspension wire of the ball (d) is not tensioned, so that the ball (d) is in a weightless state.

Grid 8. Spaceship to the Sun

In a future program, NASA plans to launch a spacecraft, aimed directly at the Sun, without a human crew, to gather information, on its way to the Sun, both about all the inner planets and, in particular, about the Sun.

They are known: the distance Earth - Sun, $r_{ES} = 1.5 \cdot 10^{11}$ m; the period of the Earth's rotation around the Sun, $T_E = 3.15 \cdot 10^7$ s; the radius of the Sun, $R_S \approx \frac{r_{ES}}{200}$.

If the launch of the spacecraft will be done in such a way that its motion relative to the Sun is a free fall, then the approximate duration of the Earth-Sun flight is:

a) $t \approx 3{,}6 \cdot 10^5$ s; b) $t \approx 6{,}6 \cdot 10^6$ s; **c)** $t \approx 5{,}6 \cdot 10^6$ s; d) $t \approx 4{,}6 \cdot 10^6$ s.

Solution

Imagine that a spacecraft is launched from the Earth whose speed in relation to the Sun is slightly lower than the orbital speed of the Earth in relation to the Sun. The ship will evolve around the Sun in an elliptical trajectory, with the Sun in the focus opposite the launch point. The lower this initial speed of the ship, the longer the ellipse on which the spacecraft will evolve around the Sun will be longer. For a certain value of this speed, the spacecraft, in its elliptical motion around the Sun, will touch the surface of the Sun, as shown in Figure 1. In this case, the height of the ship's orbit is equal to the Sun's radius. which we know is:

$$r_{min} = R_S \approx \frac{r_{ES}}{200}.$$

As a result, the semi-axes of the ship's elliptical orbit, tangent to the Sun's surface, are a and b << a, respectively, so that the small half-axis of this elliptical orbit can be neglected ($b \approx 0$). Under these conditions, the trajectory of the ship in relation to the Sun can be estimated to be rectilinear, joining the launching point of the ship with the Sun, so that the ship will reach the surface of the Sun, in point B, after a free fall towards it.

Conclusion: In order for the ship to fall freely on the surface of the Sun, in point B, after a rectilinear course in relation to it, at the initial moment of its launch from the surface of the Earth, the speed of the ship in relation to the Sun must have been zero.

To meet this condition, it is necessary that at the time of launching the ship from the Earth's surface, a speed relative to the Earth be imprinted on the ship, so that the speed of the ship relative to the Sun is zero.

The Earth's launch velocity relative to Earth must be inversely related to the orientation of the Earth's orbital velocity relative to the Sun, as shown in Figure b in Figure 1, and their modulus must be equal:

$$v_{ES} = \frac{2\pi r_{ES}}{T_E} = \frac{2 \cdot 3{,}14 \cdot 1{,}5 \cdot 10^{11}\ \text{m}}{3{,}15 \cdot 10^7\ \text{s}} \approx 3 \cdot 10^4\ \text{m/s} = 30\ \text{km/s}.$$

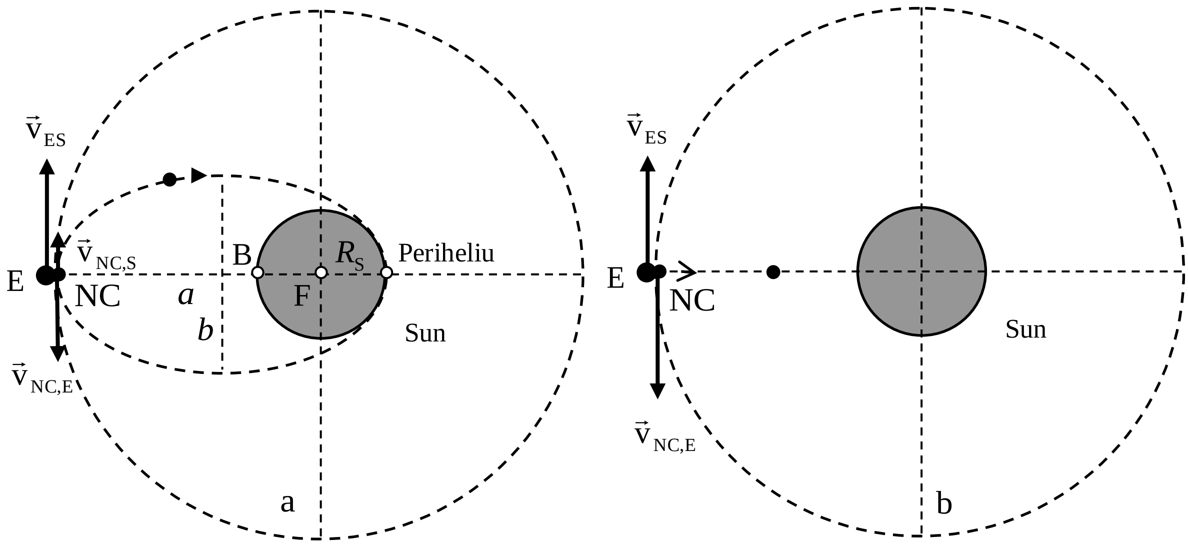


Fig. 1

According to Kepler's third law, it can be written that:

$$\frac{T_E^2}{r_{ES}^3} = \frac{T_{NC}^2}{\left(\frac{r_{ES}}{2}\right)^3},$$

where T_{NC} is the period of the spacecraft's motion on the elongated elliptical trajectory around the Sun;

$$T_{NC} = \frac{T_E}{2\sqrt{2}},$$

so that the duration of the free fall of the ship on the surface of the Sun is:

$$t = \frac{T_{NC}}{2} = \frac{T_E}{4\sqrt{2}} \approx 5.6 \cdot 10^6 \text{ s.}$$

Grid 9. The Moon, seen on the way to the Moon

The Moon, whose linear diameter is $d = 3\,436\ \text{km}$, at Earth in the distance $r_0 = 348\,000\ \text{km}$, has the apparent magnitude $m_0 = -12.7$. The apparent magnitude of the Sun, seen from Earth, is $m_s = -26.84$.

The distance, r, from the Moon, where a cosmonaut is, on the way of his spaceship to the Moon, the brightness of the Moon should be the same as the brightness of the Sun seen from Earth, is:

a) $r \approx 675.25\ \text{km}$; **b)** $r \approx 568.88\ \text{km}$; c) $r \approx 763.23\ \text{km}$; d) $r \approx 837.44\ \text{km}$.

Solution

The approach of the cosmonaut to the Moon must be such that it determines a variation of the apparent magnitude of the Moon:

$$\Delta m = m - m_0 = m_S - m_0 = -26.84 - (-12.7) = -14.14,$$

so that the apparent magnitude of the Moon, viewed from this new distance, is:

$$m = m_0 + \Delta m = m_S = -26.84;$$

$$\Delta m = m_S - m_0 = -26.84 + 12.7 = -14.14.$$

Knowing that the luminous intensity, I, of a star, in particular of the Moon, is:

$$I = \frac{\phi}{\Omega} = \frac{L}{\Omega} = \frac{W}{t \cdot \Omega},$$

that is, the energy flow emitted by the star, the Moon, in the unit of solid angle, then:

$$\Omega = \frac{A}{\Delta^2};\ A = \Omega \cdot \Delta^2;\ \phi = I \cdot \Omega;\ E = \frac{\phi}{A} = \frac{I \cdot \Omega}{\Omega \cdot \Delta^2} = \frac{I}{\Delta^2};\ E = \frac{L}{4\pi\Delta^2}.$$

According to Pogson's formula, if E_0 and respectively E are the brightnesses of the Moon, corresponding to the initial (r_0) and respectively the final distances (r), shown in the drawing in figure 1, it results:

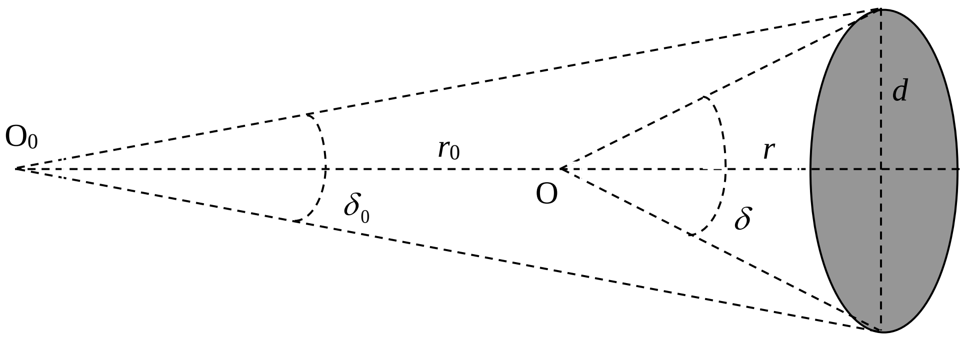


Fig. 1

$$\log \frac{E}{E_0} = -0.4 \cdot (m - m_0) = \log 10^{-0.4(m - m_0)};\ E_0 = \frac{I}{r_0^2};\ E = \frac{I}{r^2},$$

where I is the luminous intensity of the Moon;

$$\log \frac{r_0^2}{r^2} = \log 10^{-0.4(m - m_0)};$$

$$2 \log \frac{r_0}{r} = -0.4 \cdot \Delta m;\ \log \frac{r_0}{r} = 0.2 \cdot 14.14 = 2.828 = \log 10^{2.828};$$

$$\frac{r_0}{r} = 10^{2.828};\ r = \frac{r_0}{10^{2.828}}$$

$$x = 10^{2.828}; \quad \log x = 2.828; \quad x \approx 675;$$

$$r = \frac{r_0}{10^{2.828}}; \; 10^{2.828} \approx 675;$$

$$r = \frac{r_0}{675} = \frac{384000 \text{ km}}{675} \approx 568.88 \text{ km},$$

this is the distance from which the Moon should be viewed, so that its brightness is the same as the brightness of the Sun, seen from Earth.

Grid 10. The brightnesses of a star

The luminosity of a star Σ, is $L_\Sigma = 100 \cdot L_S$, where L_S is the luminosity of the Sun, and the star's surface temperature is $T_\Sigma = \frac{1}{2} \cdot T_S$, where T_S is the Sun's surface of the Sun.

Relation between the radius of the star and the radius of the Sun is:

a) $\frac{R_\Sigma}{R_S} = 40$; b) $\frac{R_\Sigma}{R_S} = 30$; c) $\frac{R_\Sigma}{R_S} = 60$; d) $\frac{R_\Sigma}{R_S} = 50$.

Solution

The luminosity of a star, meaning the energy of the total radiation emitted by that star, in the unit of time, through its entire surface, at all wavelengths, in accordance with the Stefan-Boltzmann law, results:

$$L_\Sigma = 100 \cdot L_S;$$

$$L_\Sigma = 4\pi \cdot R_\Sigma^2 \cdot \sigma \cdot T_\Sigma^4; \; L_S = 4\pi \cdot R_S^2 \cdot \sigma \cdot T_S^4;$$

$$4\pi \cdot R_\Sigma^2 \cdot \sigma \cdot T_\Sigma^4 = 10^2 \cdot 4\pi \cdot R_S^2 \cdot \sigma \cdot T_S^4; \; R_\Sigma^2 \cdot T_\Sigma^4 = 10^2 \cdot R_S^2 \cdot T_S^4;$$

$$\frac{R_\Sigma^2}{R_S^2} = 10^2 \cdot \frac{T_S^4}{T_\Sigma^4} = 10^2 \cdot \left(\frac{T_S}{T_\Sigma}\right)^4 = 10^2 \cdot 2^4;$$

$$\frac{R_\Sigma}{R_S} = 40.$$

Problem 1. Zodiac IOAA – J - 2022

In their apparent motions, the Moon and the other large planets in our solar system do not stray far from the plane of the ecliptic. Their apparent trajectories described on the celestial sphere remain contained in a region that extends symmetrically on both sides of the ecliptic, having a total width of about 18º. In the drawing in figure 1, in the plane of the ecliptic, the heliocentric orbit of the Earth is shown, and in the drawing in figure 2, the equivalent apparent geocentric orbit of the Sun, in relation to the Earth, is shown.

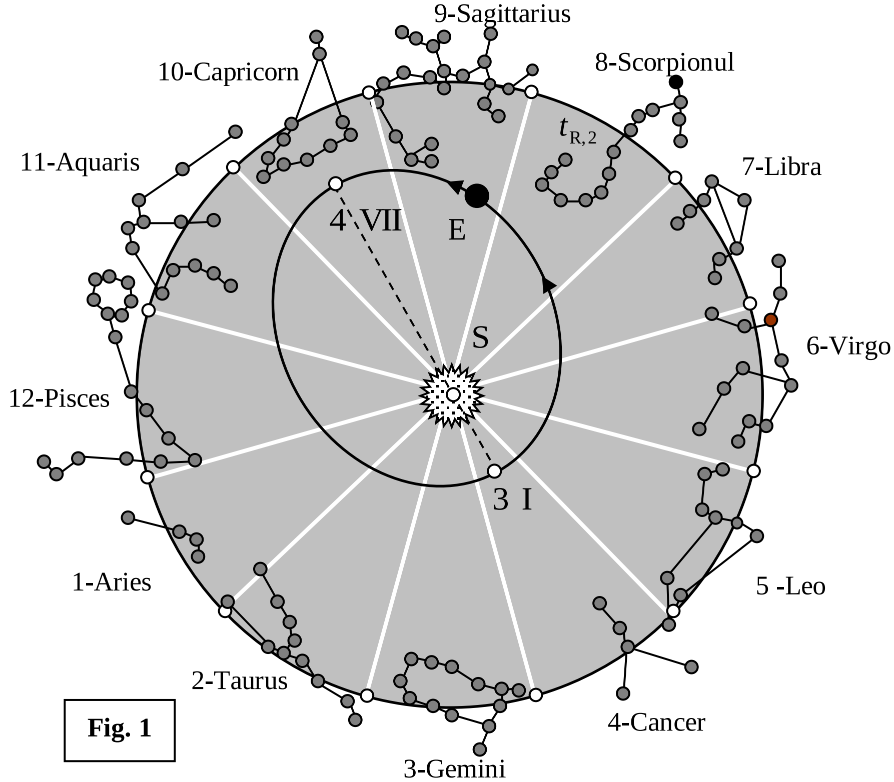


Fig. 1

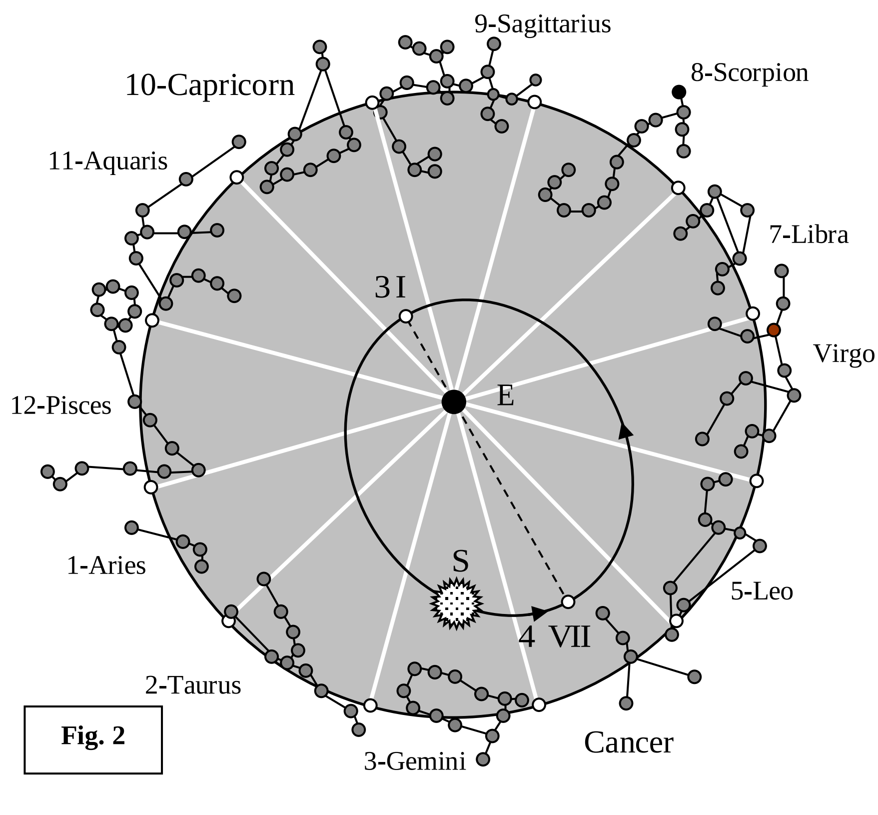


Fig. 2

The time intervals of the evolution of the true Sun in each of the 12 constellations of the Zodiac are as follows: **1)** Aries, 21 III - 20 IV; **2)** Taurus, 21 IV - 20 V; **3)** Gemini, 21 V - 20 VI; **4)** Cancer, 21 VI - 22 VII; **5)** Leo, 23 VII - 22 VIII; **6)** Virgo, 23 VIII - 22 IX; **7)** Libra, 23 IX - 22 X; **8)** Scorpio, 23 X - 21 XI; **9)** Sagittarius, 22 XI - 21 XII; **10)** Capricorn, 22 XII - 21 I; **11)** Aquarius, 22 I - 19 II; **12)** Pisces, 20 II - 21 III.

The International Olympics of Astronomy and Astrophysics for Juniors, Edition I, was to take place in SUCEAVA, in ROMANIA, on March 28 - April 3, 2020. The event, proposed by ROMANIA, was established at the International Olympics of Astronomy and Astrophysics, 13th Edition, held in Hungary, on August 2-10, 2019.

But, given the known conditions, OIAA for Juniors, Edition I, is taking place in Romania, only now, in November 2022!

a) *Identify*:

1. the sign of the OIAA development, 13th Edition, Hungary, August 2 - 10, 2019;

2. the sign when the OIAA - J was to take place - Edition I, March 28 - April 3, 2020, Romania (Edition I, proposal);

3. the sign of the OIAA - J - Edition I, November, 2022.

b) Each of the 3 specified Olympic events can be considered to have taken place on the day when the Sun, viewed from Earth, was in the middle of the angular range corresponding to each of the 3 signs.

1. *Determine* the time interval between any two of these three events:

$$\Delta t_{\mathrm{H-R,1}};\ \Delta t_{\mathrm{R,1-R,2}};\ \Delta t_{\mathrm{H-R,2}}.$$

2. *Estimate*, by direct measurements on the drawing, by means of a protractor, the value of the angle between the position vector of the Earth, in relation to the Sun, corresponding to each of the three specified moments, ie the directions of the vectors $\vec{r}_{\mathrm{H}}$, $\vec{r}_{\mathrm{R,1}}$ and respectively $\vec{r}_{\mathrm{R,2}}$, and the direction of the apse line, Aph – Ph, ie the angles α_{H}, $\alpha_{\mathrm{R,1}}$ and respectively $\alpha_{\mathrm{R,2}}$.

c) *Determine* the area of the surface described by the position vector of the center of the Earth, $\vec{r}$, in relation to the center of the Sun:

1. from position $\vec{r}_{\mathrm{Hungry}}$ to position $\vec{r}_{\mathrm{Romania,1}}$;

2. from position $\vec{r}_{\mathrm{Romania,1}}$ to position $\vec{r}_{\mathrm{Romania,2}}$;

3. from position $\vec{r}_{\mathrm{Hungry}}$ to position $\vec{r}_{\mathrm{Romania,2}}$.

d) *Determine*:

1. the distance between the Earth and the Sun, on the day when in Suceava, in 2020, the IOAA for Juniors, Edition I should have taken place, $r_{\mathrm{R,1}}$, if, for the ellipse representing the Earth's orbit around the Sun, the following are known: large semiaxis, $a = 149\,597\,500\ \mathrm{km}$ and small semiaxis, $b = 149\,580\,670\ \mathrm{km}$;

2. the acceleration of the center of the Earth, on the day when in Suceava, in 2020, the IOAA for Juniors should have taken place, $a_{\mathrm{E,1}}$, and compared with the gravitational acceleration in the gravitational field of the Sun, corresponding to the distance $r_{\mathrm{R,1}}$ from the center of the Sun, $g_{\mathrm{S,1}}$;

3. the components of the speed of the center of the Earth, $\vec{\mathrm{v}}_{\mathrm{R}}$, parallel to the major axis of the ellipse and respectively perpendicular to the major axis of the ellipse, in the days of the International Olympics in Romania, $\mathrm{v}_{//}$ and respectively $\mathrm{v}_{\perp}$.

It is known that the kinetic moment of the Earth in relation to the center of the Sun and the total mechanical energy of the Earth-Sun system are given by the expressions:

$$L = M_{\mathrm{E}} b \cdot \sqrt{\frac{K M_{\mathrm{S}}}{a}};\ E = -K \frac{M_{\mathrm{E}} M_{\mathrm{S}}}{2a}.$$

Given: gravitational attraction constant, $K = 6.67 \cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}$; mass of the Sun, $M_{\mathrm{S}} = 1.989 \cdot 10^{30}\ \mathrm{kg}$.

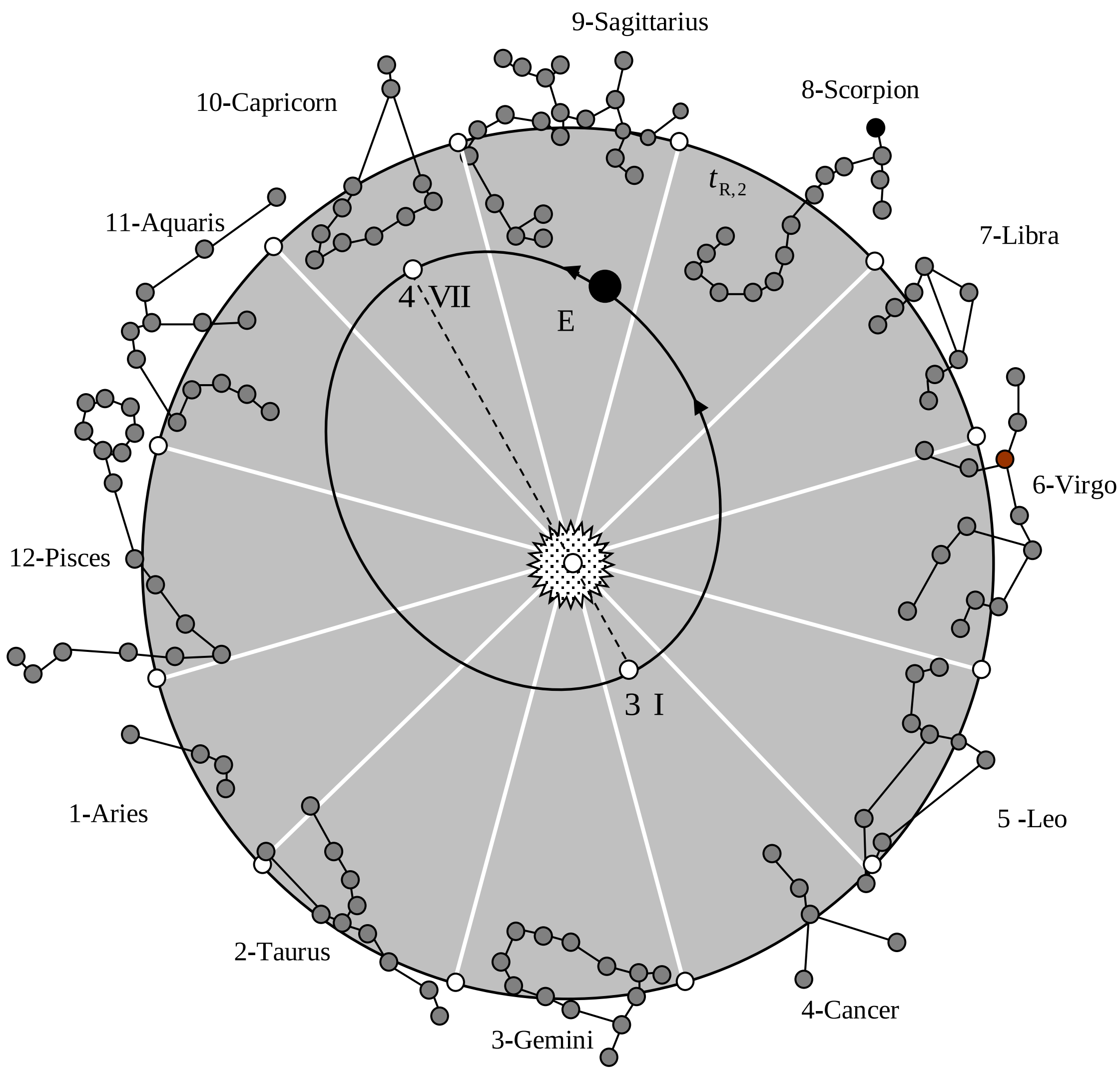


Figure 1, reproduced in enlarged format!
This image will be worked on and will be handed in with the competition sheets!

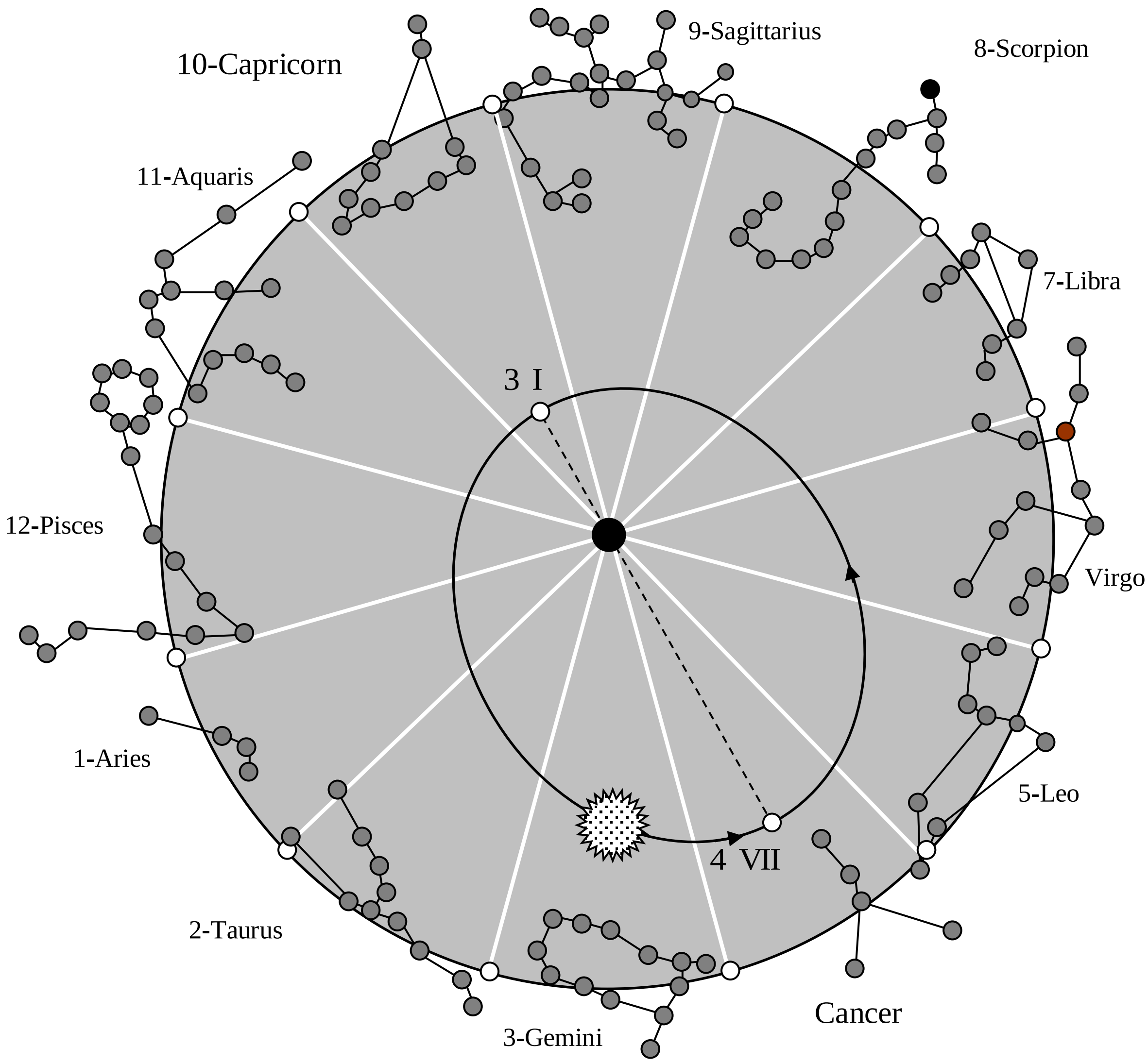


Figure 2, reproduced in enlarged format!
This image will be worked on and will be handed in with the competition sheets!

Solving

a)

1) In the drawing in figure 3 are represented the positions of the Earth on its elliptical orbit around the Sun, as well as the positions of the projections of the Sun in the zodiacal belt, corresponding to the moments of the three events, specified in the statement of the problem.

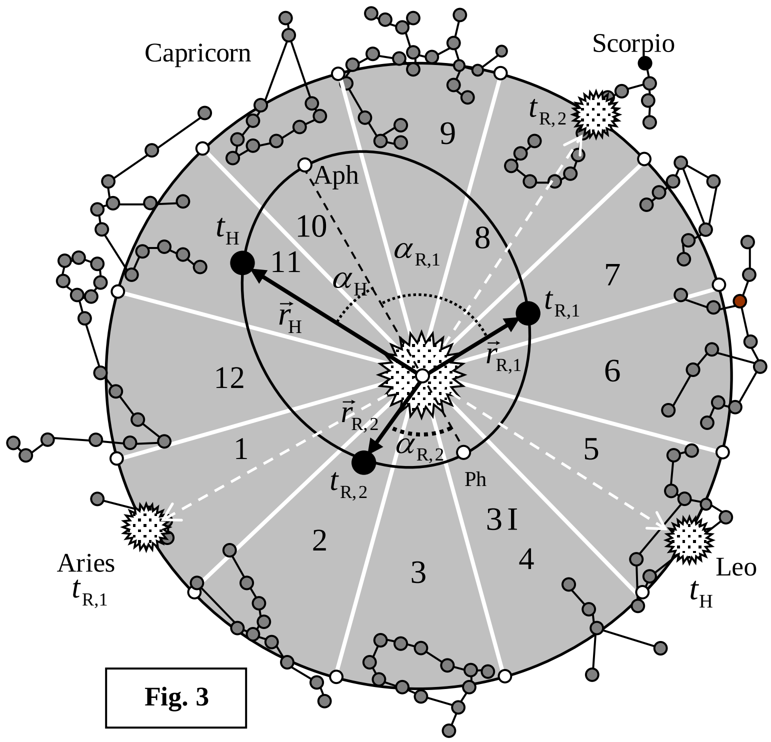


Fig. 3

In the drawing in figure 4, the positions of the Sun on its apparent elliptical orbit around the Earth are represented, as well as the positions of the projections of the Sun in the zodiac belt, corresponding to the moments of the three events, specified in the statement of the problem.

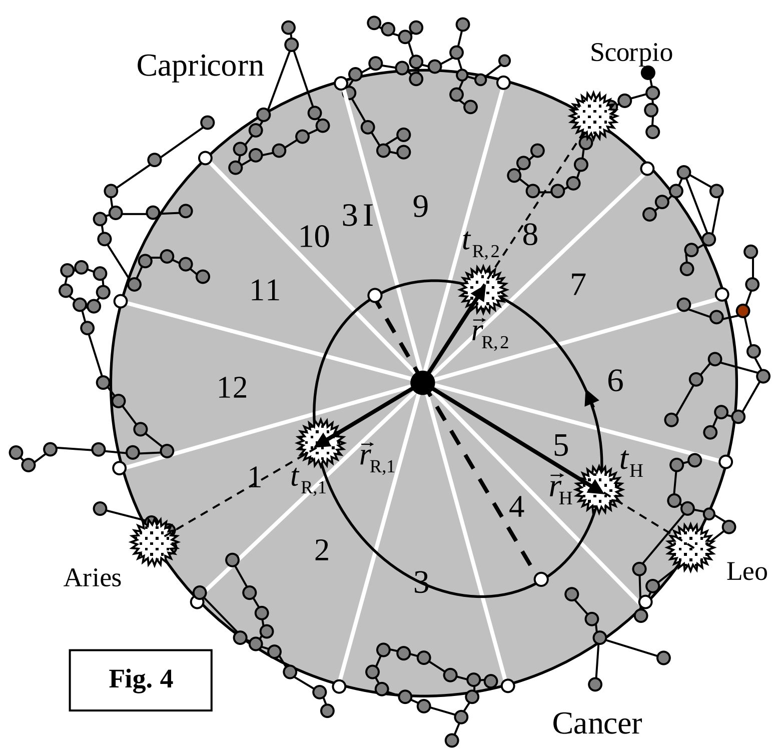


Fig. 4

In these conditions, we identify:

1. the announcement from Hungary was made in the sign of LEO, in the middle of it, on August 6, 2019;

2. the event proposed to take place in Romania was to take place in the sign of ARIES, in the middle of it, on April 4, 2020;

3. the International Olympics of Astronomy and Astrophysics, for Juniors, Edition I, takes place in Romania, in November, 2022, in the sign of SCORPIO.

b) The position vectors of the Earth, in relation to the Sun, as well as the moments corresponding to the three events, are:

1. for IOAA – 2019, Hungary: $\vec{r}_{\mathrm{H}}$; $t_{\mathrm{H}} =$ 6 August, 2019;

2. for IOAA – J – 2020, Romania - proposal: $\vec{r}_{\mathrm{R},1}$; $t_{\mathrm{R},1} =$ 4 April, 2020,

3. for IOAA – J – 2022, Romania: $\vec{r}_{\mathrm{R},2}$; $t_{\mathrm{R},2} =$

so that the time intervals between any two of these three events are:

$$\Delta t_{\mathrm{H-R},1} = t_{\mathrm{R},1} - t_{\mathrm{H}} = 223 \text{ days};$$

$$\Delta t_{\mathrm{R},1-\mathrm{R},2} = t_{\mathrm{R},2} - t_{\mathrm{R},1} = \text{ days};$$

$$\Delta t_{\mathrm{H-R},2} = t_{\mathrm{R},2} - t_{\mathrm{H}} = \Delta t_{\mathrm{H-R},1} + \Delta t_{\mathrm{R},1-\mathrm{R},2} = 223 \text{ days} + 886 \text{ days} = 1109 \text{ days}.$$

From direct measurements, performed on the drawing in Figure 3, it results:

$$\alpha_{\mathrm{H}} \approx 30^0;$$

$$\alpha_{\mathrm{R},1} \approx 90^0;\ \vec{r}_{\mathrm{R},1} \perp (\mathrm{Aph-Ph});$$

$$\alpha_{\mathrm{R},2} \approx 60^0.$$

c)

1. It is known that in the evolution of the Earth in the elliptical orbit around the Sun, according to Keppler's third law, the areolar velocity of the Earth is constant, so that:

$$\Omega = \frac{\Delta A}{\Delta t} = \text{constant};$$

$$\frac{A_{\text{ellipse}}}{T} = \frac{\Delta A_{\mathrm{H-R},1}}{\Delta t_{\mathrm{H-R},1}};$$

$$\Delta A_{\mathrm{H-R},1} = \frac{\Delta t_{\mathrm{H-R},1}}{T} A_{\text{ellipse}} = \frac{\Delta t_{\mathrm{H-R},1}}{T} \cdot \pi \cdot a \cdot b;$$

$$\Delta t_{\mathrm{H-R},1} = 223 \text{ days};\ T = 365.256 \text{ days};$$

$$a = 149\,597\,500 \text{ km};\ b = 149\,580\,670 \text{ km};$$

$$\Delta A_{\mathrm{H-R},1} = \frac{223 \text{ days}}{365.256 \text{ days}} \cdot 3.14 \cdot 149\,597\,500 \text{ km} \cdot 149\,580\,670 \text{ km};$$

$$\Delta A_{\mathrm{H-R},1} = \frac{223}{365.256} \cdot 3.14 \cdot 149\,597\,500 \cdot 149\,580\,670 \text{ km}^2;$$

$$\Delta A_{\mathrm{H-R},1} = 4.2897 \cdot 10^{16} \text{ km}^2.$$

2.

$$\frac{A_{\text{ellipse}}}{T} = \frac{\Delta A_{\text{R},1-\text{R},2}}{\Delta t_{\text{R},1-\text{R},2}};$$

$$\Delta A_{\text{R},1-\text{R},2} = \frac{\Delta t_{\text{R},1-\text{R},2}}{T} A_{\text{ellipse}} = \frac{\Delta t_{\text{R},1-\text{R},2}}{T} \cdot \pi \cdot a \cdot b;$$

$$\Delta A_{\text{R},1-\text{R},2} = \frac{886\text{ days}}{365.256\text{ days}} \cdot 3.14 \cdot 149\,597\,500\text{ km} \cdot 149\,580\,670\text{ km};$$

$$\Delta A_{\text{R},1-\text{R},2} = \frac{886}{365.256} \cdot 3.14 \cdot 149\,597\,500 \cdot 149\,580\,670\text{ km}^2;$$

$$\Delta A_{\text{R},1-\text{R},2} = 1.704 \cdot 10^{17}\text{ km}^2;$$

3.

$$\frac{A_{\text{ellipse}}}{T} = \frac{\Delta A_{\text{H}-\text{R},2}}{\Delta t_{\text{H}-\text{R},2}}; \quad \frac{A_{\text{ellipse}}}{T} = \frac{\Delta A_{\text{H}-\text{R},1}}{\Delta t_{\text{H}-\text{R},1}};$$

$$\Delta A_{\text{H}-\text{R},2} = \frac{\Delta t_{\text{H}-\text{R},2}}{T} A_{\text{ellipse}} = \frac{\Delta t_{\text{H}-\text{R},2}}{T} \cdot \pi \cdot a \cdot b;$$

$$\Delta t_{\text{H}-\text{R},2} = 1109\text{ days}; \; T = 365.256\text{ days};$$

$$a = 149\,597\,500\text{ km}; \quad b = 149\,580\,670\text{ km};$$

$$\Delta A_{\text{H}-\text{R},2} = \frac{1109\text{ days}}{365.256\text{ days}} \cdot 3.14 \cdot 149\,597\,500\text{ km} \cdot 149\,580\,670\text{ km};$$

$$\Delta A_{\text{H}-\text{R},2} = \frac{1109}{365.256} \cdot 3.14 \cdot 149\,597\,500 \cdot 149\,580\,670\text{ km}^2;$$

$$\Delta A_{\text{H}-\text{R},2} = 2.133 \cdot 10^{17}\text{ km}^2.$$

d)

1. According to the notations in figure 5, representing the position of the Earth, in relation to the Sun, on the day of, when, as we have shown, $\vec{r}_{\text{R},1} \perp (\text{Aph} - \text{Ph})$, it results:

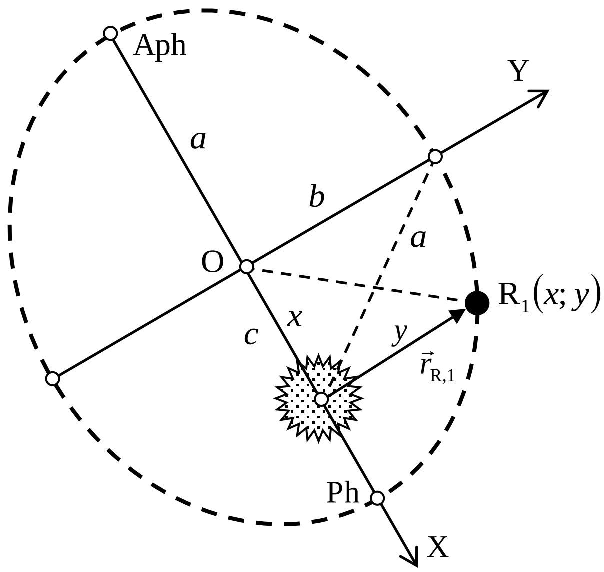


Fig. 5

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1;\ \ x=c;\ y=r_{R,1};$$

$$c^2=a^2-b^2;$$

$$\frac{c^2}{a^2}+\frac{r_{R,1}^2}{b^2}=1;\ \frac{a^2-b^2}{a^2}+\frac{r_{R,1}^2}{b^2}=1;$$

$$1-\frac{b^2}{a^2}+\frac{r_{R,1}^2}{b^2}=1;\ \frac{r_{R,1}^2}{b^2}=\frac{b^2}{a^2};$$

$$r_{R,1}^2=\frac{b^4}{a^2};$$

$$r_{R,1}=\frac{b^2}{a},$$

representing the distance between the center of the Earth and the center of the Sun on the day when the International Olympiad in Astronomy and Astrophysics should have taken place, for Juniorim Edition I, on March 28 - April 3, 2020, in Romania;

$$a=149\,597\,500\ \text{km};\ \ b=149\,580\,670\ \text{km};$$

$$r_{R,1}=149\,563\,841.9\ \text{km}.$$

2.

$$K\frac{M_E M_S}{r_{R,1}^2}=M_E a_{E,1};\ a_{E,1}=K\frac{M_S}{r_{R,1}^2};$$

$$K=6.67\cdot10^{-11}\ \text{Nm}^2\text{kg}^{-2};\ M_S=1.989\cdot10^{30}\ \text{kg};$$

$$a_{E,1}=\frac{6.67\cdot10^{-11}\cdot1.989\cdot10^{30}}{(149\,563\,841.9)^2\cdot10^6}\ \frac{\text{m}}{\text{s}^2};$$

$$a_{E,1}\approx0.006\,\frac{\text{m}}{\text{s}^2}.$$

$$g_{S,1} = K \cdot \frac{M_S}{r_{R,1}^2} = a_{E,1}.$$

3. Corresponding to the position R_1 of the Earth, represented in the drawing in Figure 6, when its kinetic moment is:

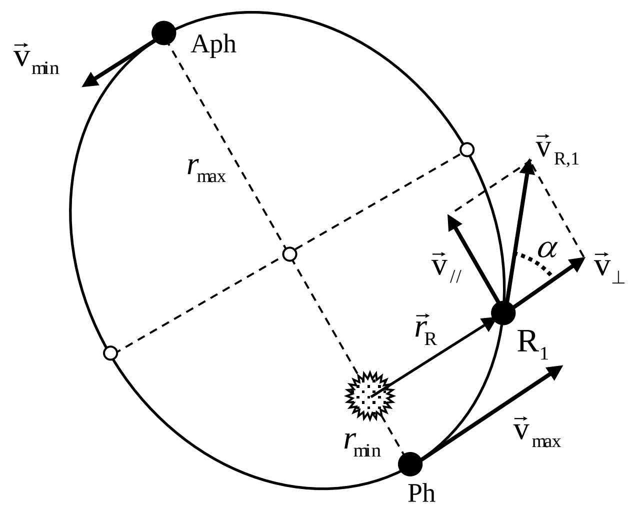


Fig. 6

$$\vec{L}_{R,1} = \vec{r}_{R,1} \,\mathrm{x}\, M_E \,\vec{v}_{R,1};\; L_{R,1} = r_{R,1} M_E \,v_{R,1} \cdot \sin\left(\vec{r}_{R,1}; \vec{v}_{R,1}\right) = r_{R,1} M_E \,v_{R,1} \cdot \sin\alpha,$$

according to the law of conservation of kinetic momentum, it follows:

$$v_{R,1} \cdot \sin\alpha = v_{//};$$

$$L_{R,1} = r_{R,1} M_E \,v_{//};\; L = M_E b \cdot \sqrt{\frac{KM_S}{a}};$$

$$M_E b \cdot \sqrt{\frac{KM_S}{a}} = r_{R,1} M_E \,v_{//};\; r_R = \frac{b^2}{a};$$

$$v_{//} = \frac{a}{b} \cdot \sqrt{\frac{KM_S}{a}};$$

$$K = 6.67 \cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}};\; M_S = 1.989 \cdot 10^{30}\ \mathrm{kg};$$

$$a = 149\,597\,500\ \mathrm{km};\;\; b = 149\,580\,670\ \mathrm{km};$$

$$v_{//} = \frac{149\,597\,500\ \mathrm{km}}{149\,580\,670\ \mathrm{km}} \cdot \sqrt{\frac{6.67 \cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}} \cdot 1.989 \cdot 10^{30}\ \mathrm{kg}}{149\,597\,500 \cdot 10^3\ \mathrm{m}}};$$

$$v_{//} = \frac{149\,597\,500}{149\,580\,670} \cdot \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 1.989 \cdot 10^{30}}{149\,597\,500 \cdot 10^3}}\ \frac{\mathrm{m}}{\mathrm{s}};$$

$$v_{//} = \frac{149\,597\,500}{149\,580\,670} \cdot \sqrt{\frac{6.67 \cdot 1.989}{149\,597\,500}} \cdot 10^8\ \frac{\mathrm{m}}{\mathrm{s}};$$

$$v_{//} = \frac{\sqrt{149\,597\,500 \cdot 6.67 \cdot 1.989}}{149\,580\,670} \cdot 10^8\ \frac{\mathrm{m}}{\mathrm{s}};$$

$$\mathrm{v}_{//} = 29\,782.9\,\frac{\mathrm{m}}{\mathrm{s}};\ \mathrm{v}_{//} = 29\,782.9\cdot 10^{-3}\,\frac{\mathrm{km}}{\mathrm{s}};$$

$$\mathrm{v}_{//} = 29.7829\,\frac{\mathrm{km}}{\mathrm{s}}.$$

According to the law of conservation of mechanical energy it results:

$$E = -K\frac{M_{\mathrm{E}}M_{\mathrm{S}}}{2a};$$

$$E_{\mathrm{R,1}} = \frac{M_{\mathrm{E}}\,\mathrm{v}_{\mathrm{R}}^2}{2} - K\frac{M_{\mathrm{E}}M_{\mathrm{S}}}{r_{\mathrm{R,1}}};$$

$$\frac{M_{\mathrm{E}}\,\mathrm{v}_{\mathrm{R,1}}^2}{2} - K\frac{M_{\mathrm{E}}M_{\mathrm{S}}}{r_{\mathrm{R,1}}} = -K\frac{M_{\mathrm{E}}M_{\mathrm{S}}}{2a};$$

$$\frac{\mathrm{v}_{\mathrm{R,1}}^2}{2} - K\frac{M_{\mathrm{S}}}{r_{\mathrm{R,1}}} = -K\frac{M_{\mathrm{S}}}{2a};$$

$$\mathrm{v}_{\mathrm{R,1}} = \sqrt{KM_{\mathrm{S}}\left(\frac{2}{r_{\mathrm{R,1}}} - \frac{1}{a}\right)} = \sqrt{KM_{\mathrm{S}}\cdot\frac{2a - r_{\mathrm{R,1}}}{r_{\mathrm{R,1}}a}};$$

$$K = 6.67\cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}};\ M_{\mathrm{S}} = 1.989\cdot 10^{30}\ \mathrm{kg};\ r_{\mathrm{R,1}} = 149\,563\,841{,}9\ \mathrm{km};\ a = 149\,597\,500\ \mathrm{km};$$

$$\mathrm{v}_{\mathrm{R,1}} = \sqrt{6.67\cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}\cdot 1.989\cdot 10^{30}\ \mathrm{kg}\cdot\frac{2\cdot 149\,597\,500\ \mathrm{km} - 149\,563\,841.9\ \mathrm{km}}{149\,563\,841.9\ \mathrm{km}\cdot 149\,597\,500\ \mathrm{km}}};$$

$$\mathrm{v}_{\mathrm{R,1}} = \sqrt{6.67\cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}\cdot 1.989\cdot 10^{30}\ \mathrm{kg}\cdot\frac{2\cdot 149\,597\,500 - 149\,563\,841.9}{149\,563\,841.9\cdot 149\,597\,500\cdot 10^3\ \mathrm{m}}};$$

$$\mathrm{v}_{\mathrm{R,1}} = \sqrt{6.67\cdot 10^{-11}\cdot 1.989\cdot 10^{30}\cdot\frac{2\cdot 149\,597\,500 - 149\,563\,841.9}{149\,563\,841.9\cdot 149\,597\,500\cdot 10^3}\cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}};$$

$$\mathrm{v}_{\mathrm{R,1}} = 29\,786.25\,\frac{\mathrm{m}}{\mathrm{s}} = 29\,786.25\cdot 10^{-3}\,\frac{\mathrm{km}}{\mathrm{s}};$$

$$\mathrm{v}_{\mathrm{R,1}} = 29.7862\,\frac{\mathrm{km}}{\mathrm{s}};$$

$$\mathrm{v}_{//} = 29.7829\,\frac{\mathrm{km}}{\mathrm{s}};$$

$$\mathrm{v}_{\perp} = \sqrt{\mathrm{v}_{\mathrm{R,1}}^2 - \mathrm{v}_{//}^2} = 0.4433\,\frac{\mathrm{km}}{\mathrm{s}}.$$

Problem 2. The rocket that destroys the threatening asteroid

With the help of a rocket, R, launched from a spaceship, N, an asteroid, A, must be destroyed, approaching the Earth threateningly, coming right in the direction of the center of the Earth, as shown in the drawing in Figure 1. There are positions of the ship spacecraft, N and the asteroid A, respectively, at the time of the launch of the rocket R, to meet the asteroid A, when the speed of the spacecraft, N, relative to Earth, is constant,, oriented along the spacecraft line - asteroid, and the speed of asteroid A, relative to Earth, is considered constant, $\vec{v}_N$, oriented along the spacecraft line - asteroid, and the speed of asteroid A, relative to Earth, is considered constant, $\vec{v}_A$, its direction forming an angle α with the direction of the spacecraft, N.

During the flight of the rocket, from its launch to the impact with the asteroid, it is considered that the Earth, relative to the Sun, is at rest, and the movements of the rocket, R, the spacecraft, N, and the asteroid, A, are rectilinear and uniform movements.

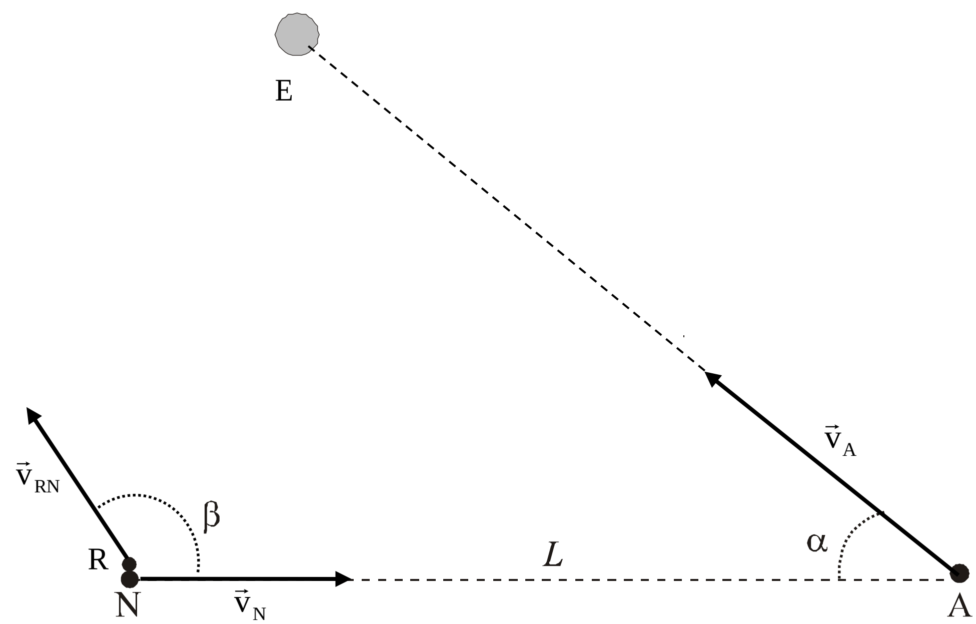


Fig. 1

Knowing that the direction of the rocket launching R was chosen in such a way that the duration of the rocket's movement, R, from launch to impact with asteroid A, is minimal, *determine* the angle β, at which the rocket R was launched, relative to the direction of movement spacecraft, N;

Solving

Reaching the target, asteroid A, by the rocket R, is done in a minimum time, if the direction of absolute displacement of the rocket R, ie the direction of movement NC, is perpendicular to the direction of absolute movement of asteroid A, ie the direction of movement AC, as indicated the drawing in Figure 2, which shows:

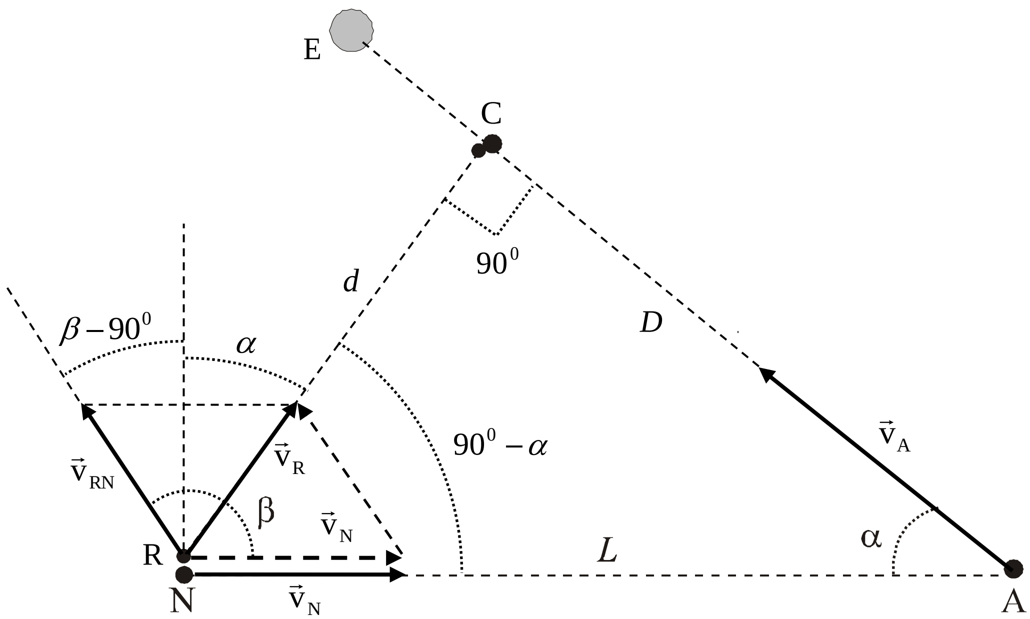


Fig. 2

$$D = v_A \cdot t_{min} = L \cdot \cos\alpha;$$
$$d = v_R \cdot t_{min} = L \cdot \sin\alpha,$$

where v_R is the absolute speed of the R rocket, from the moment of its launch;

$$t_{min} = \frac{L\cdot\cos\alpha}{v_A};\ t_{min} = \frac{L\cdot\sin\alpha}{v_R};$$
$$\frac{L\cdot\cos\alpha}{v_A} = \frac{L\cdot\sin\alpha}{v_R};\ \frac{\cos\alpha}{v_A} = \frac{\sin\alpha}{v_R};$$
$$v_R = v_A \cdot \tan\alpha,$$

where v_R is the absolute speed of the rocket, after launch;

$$\vec{v}_{RN} = \vec{v}_R - \vec{v}_A,$$

where $\vec{v}_{RN}$ is the relative speed of the rocket R, relative to the spacecraft N;

$$v_{RN}^2 = v_R^2 + v_A^2 - 2\cdot v_R \cdot v_N \cdot \cos(90^0 - \alpha);$$
$$v_{RN}^2 = v_R^2 + v_A^2 - 2\cdot v_R \cdot v_N \cdot \sin\alpha;$$
$$v_R \cdot \cos\alpha = v_{RN} \cdot \cos(\beta - 90^0),$$

where β is the angle at which the rocket R is launched from the spacecraft, N, relative to the flight direction of the spacecraft, N;

$$v_R \cdot \cos\alpha = v_{RN} \cdot \sin\beta;$$

$$\sin\beta = \frac{v_R}{v_{RN}} \cdot \cos\alpha;$$

$$v_R = v_A \cdot \tan\alpha;$$

$$\sin\beta = \frac{v_A \cdot \tan\alpha}{v_{RN}} \cdot \cos\alpha = \frac{v_A}{v_{RN}} \cdot \frac{\sin\alpha}{\cos\alpha} \cdot \cos\alpha;$$

$$\sin\beta = \frac{v_A}{v_{RN}} \cdot \sin\alpha;$$

$$v_{RN}^2 = v_R^2 + v_A^2 - 2 \cdot v_R \cdot v_N \cdot \sin\alpha;$$

$$v_R = v_A \cdot \tan\alpha;$$

$$v_{RN}^2 = v_A^2 \cdot \frac{\sin^2\alpha}{\cos^2\alpha} + v_A^2 - 2 \cdot v_A \cdot \frac{\sin\alpha}{\cos\alpha} \cdot v_N \cdot \sin\alpha;$$

$$v_{RN}^2 = v_A^2 \cdot \left(\frac{\sin^2\alpha}{\cos^2\alpha} + 1 \right) - 2 \cdot v_A \cdot \frac{\sin^2\alpha}{\cos\alpha} \cdot v_N;$$

$$v_{RN}^2 = v_A^2 \cdot \frac{1}{\cos^2\alpha} - 2 \cdot v_A \cdot v_N \cdot \frac{\sin^2\alpha}{\cos\alpha};$$

$$v_{RN} = v_A \cdot \sqrt{\frac{1}{\cos^2\alpha} - 2 \cdot \frac{v_N}{v_A} \cdot \frac{\sin^2\alpha}{\cos\alpha}};$$

$$v_{RN} = v_A \cdot \sqrt{\frac{1}{\cos^2\alpha} - 2 \cdot \frac{v_N}{v_A} \cdot \frac{\cos\alpha \cdot \sin^2\alpha}{\cos^2\alpha}};$$

$$v_{RN} = \frac{v_A}{\cos\alpha} \cdot \sqrt{1 - 2 \cdot \frac{v_N}{v_A} \cdot \sin^2\alpha \cdot \cos\alpha},$$

representing the relative speed of the rocket R, relative to the spacecraft N;

$$\sin\beta = \frac{v_R}{v_{RN}} \cdot \cos\alpha;$$

$$v_R = v_A \cdot \tan\alpha;$$

$$v_{RN} = \frac{v_A}{\cos\alpha} \cdot \sqrt{1 - 2 \cdot \frac{v_N}{v_A} \cdot \sin^2\alpha \cdot \cos\alpha};$$

$$\sin\beta = \frac{v_A \cdot \frac{\sin\alpha}{\cos\alpha}}{\frac{v_A}{\cos\alpha} \cdot \sqrt{1 - 2 \cdot \frac{v_N}{v_A} \cdot \sin^2\alpha \cdot \cos\alpha}} \cdot \cos\alpha;$$

$$\sin\beta = \frac{\sin\alpha \cdot \cos\alpha}{\sqrt{1 - 2 \cdot \frac{v_N}{v_A} \cdot \sin^2\alpha \cdot \cos\alpha}},$$

β being the angle at which the rocket R is launched from the spacecraft, N, relative to the direction of flight of the spacecraft, N, to destroy the asteroid.