(D02) Predicting arrival times of coronal mass ejections on Earth **[60 marks]**

The Sun occasionally releases magnetized plasma, termed coronal mass ejections (CMEs), that originate from the surface of the Sun and propagate outwards. Accurate prediction of their arrival times at Earth is crucial for understanding and mitigating their potential effects on satellites orbiting the Earth. In this problem, we aim to predict the arrival times of CMEs by developing an empirical model, using the data of 10 CMEs. Throughout this problem, the distance between the Sun's surface and Earth is taken to be $214\,R_\odot$. Further, assume that the Sun is not rotating. Due to electromagnetic, gravitational and drag forces, CMEs experience a variable acceleration throughout their propagation. In the first two parts of this problem, we assume that the region between the Sun and the Earth is vacuum.

CMEs through vacuum

(D02.1) The initial velocity, u, at the solar surface ($= 1\,R_\odot$), the final velocity, v, upon reaching Earth, and the time to arrive at Earth after leaving the surface of the Sun (in hours), τ, are given for 10 CMEs in the following table.

CME Name	u ($\mathrm{km\,s^{-1}}$)	v ($\mathrm{km\,s^{-1}}$)	τ (h)
CME-A	804	470	74.5
CME-B	247	360	127.5
CME-C	523	396	103.5
CME-D	830	415	71.0
CME-E	665	400	104.5
CME-F	347	350	101.5
CME-G	446	375	99.5
CME-H	155	360	97.0
CME-I	1016	515	67.0
CME-J	683	410	54.0

(D02.1a) Calculate the average acceleration, a, for each CME in $\mathrm{m\,s^{-2}}$. **3**

Solution:
Average acceleration (in $\mathrm{m\,s^{-2}}$) is given by the following relation,

$$a = (v - u) \times 10^3 / (\tau \times 3600)$$ 0.5

2.5

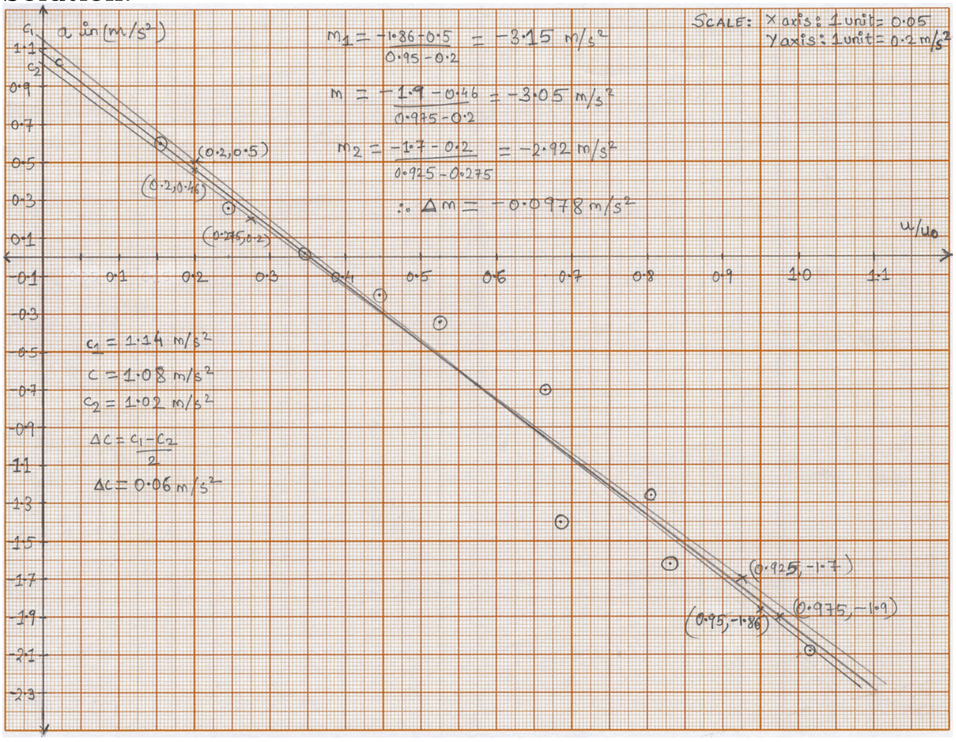
CME Name	u ($\mathrm{km\,s^{-1}}$)	v ($\mathrm{km\,s^{-1}}$)	τ (h)	a ($\mathrm{m\,s^{-2}}$)
CME-A	804	470	74.5	−1.25
CME-B	247	360	127.5	0.246
CME-C	523	396	103.5	−0.341
CME-D	830	415	71.0	−1.62
CME-E	665	400	104.5	−0.704
CME-F	347	350	101.5	0.00821
CME-G	446	375	99.5	−0.198
CME-H	155	360	97.0	0.587
CME-I	1016	515	67.0	−2.08
CME-J	683	410	54.0	−1.40

All correct values	2.5
9 correct values	2.0
7–8 correct values	1.5

5–6 correct values	1.0
3–4 correct values	0.5
$\leq$ 2 correct values	0.0

(D02.1b) We assume an empirical model for the acceleration, a_{model}, of a CME, which depends on its initial velocity, u, as $a_{\text{model}} = m\left(\frac{u}{u_0}\right) + \alpha$ where, a_{model} is expressed in $\mathrm{m\,s^{-2}}$, u is expressed in $\mathrm{km\,s^{-1}}$ and $u_0 = 1.00 \times 10^3\,\mathrm{km\,s^{-1}}$. Determine the constants m and α and their associated uncertainties using an appropriate graph (mark your graph as "D02.1b"). **15**

Solution:



Labels on a and u/u_0 axes		0.5
Either scale (including units) mentioned or ticks marked on both axes		1.0
More than 75% of the graph paper used		0.5
10 points marked correctly		2.0
8 to 9 points marked correctly	1.5	
6 to 7 points marked correctly	0.5	
$\leq$ 5 points marked correctly	0.0	
Drawing a best fit line		1.0
Computing m and α from graph		3.0
Computing errors on slope and intercept using graph paper		2.0
Correct value of m and α		3.0
Correct value of Δm and $\Delta\alpha$		2.0

$$\boxed{m = (-3.05 \pm 0.09)\,\mathrm{m\,s^{-2}}}$$

For m:

Half credit lower limit	Full credit range	Half credit upper limit
	$-2.91\,\mathrm{m\,s^{-2}}$ to $-3.19\,\mathrm{m\,s^{-2}}$	

For Δm:

Half credit lower limit	Full credit range	Half credit upper limit
	$0.06\,\mathrm{m\,s^{-2}}$ to $0.16\,\mathrm{m\,s^{-2}}$	

$\boxed{\alpha = (1.08 \pm 0.06)\,\mathrm{m\,s^{-2}}}$

For α:

Half credit lower limit	Full credit range	Half credit upper limit
	$0.99\,\mathrm{m\,s^{-2}}$ to $1.17\,\mathrm{m\,s^{-2}}$	

For $\Delta\alpha$:

Half credit lower limit	Full credit range	Half credit upper limit
	$0.04\,\mathrm{m\,s^{-2}}$ to $0.16\,\mathrm{m\,s^{-2}}$	

Thus, the expression obtained by fitting a straight line to a vs u/u_0 is

$$a_{\mathrm{model}} = -3.05\left(\frac{u}{u_0}\right) + 1.08$$

(D02.1c) For each CME, tabulate a_{model} in $\mathrm{m\,s^{-2}}$. Hence calculate the root-mean-square (rms) deviation of acceleration, δa_{rms}, between the calculated accelerations, a, and the model values, a_{model}. 4

Solution:

CME Name	a_{model} ($\mathrm{m\,s^{-2}}$)
CME-A	-1.37
CME-B	0.327
CME-C	-0.515
CME-D	-1.45
CME-E	-0.948
CME-F	0.0217
CME-G	-0.280
CME-H	0.607
CME-I	-2.02
CME-J	-1.00

2.0

9 to 10 points correctly evaluated	2.0
8 points correctly evaluated	1.0
Less than 8 points correctly evaluated	0.0

The root mean square (rms) deviation of acceleration is given by

$$\delta a_{\mathrm{rms}} = \sqrt{\frac{\sum_{i=1}^{10}(a_{\mathrm{model},i} - a_i)^2}{10}}$$ 2.0

Using this expression we get, $\boxed{\delta a_{\mathrm{rms}} = 0.176\,\mathrm{m\,s^{-2}}}$

All values where 8 or 9 or 10 was used in the denominator is acceptable.

Half credit lower limit	Full credit range	Half credit upper limit
	$0.160\,\mathrm{m\,s^{-2}}$ to $0.258\,\mathrm{m\,s^{-2}}$	

(D02.2) We consider two other CMEs: CME-1 and CME-2, with initial velocities, $u = 1044\,\mathrm{km\,s^{-1}}$ and $273\,\mathrm{km\,s^{-1}}$, respectively.

(D02.2a) Using the empirical model obtained in (D02.1b), calculate the predicted arrival times at Earth, $\tau_{1,\mathrm{m}}$ and $\tau_{2,\mathrm{m}}$ (in hours), for CME-1 and CME-2, respectively. **4**

Solution:
The time of arrival of a CME at the Earth, τ_m, can be calculated from the following relation.

$$s = u\tau_\mathrm{m} + \frac{1}{2}a_\mathrm{model}\tau_\mathrm{m}^2$$

However, this would yield two solutions for τ_m, only one of which will be for a positive final velocity (acceptable), and the other for a negative final velocity (not acceptable). Since acceleration is constant, the final velocity v_m at Earth at a distance $s = 214\,\mathrm{R_\odot}$ is given by

$$v_\mathrm{m} = \left|(u^2 + 2a_\mathrm{model}s)^{1/2}\right| = \left|\left[u^2 + 2\left(m\frac{u}{u_0} + \alpha\right)s\right]^{1/2}\right|$$ 1.0

Using this velocity, we get the arrival time τ as

$$\tau_\mathrm{m} = \frac{v_\mathrm{m} - u}{a_\mathrm{model}} = \frac{v_\mathrm{m} - u}{m\frac{u}{u_0} + \alpha}$$ 1.0

Using the given value of u, and the obtained values of m and α, we get

For CME-1:

$v_{1,\mathrm{m}} = 680.728\,\mathrm{km\,s^{-1}}$ which gives $\boxed{\tau_{1,\mathrm{m}} = 48.0\,\mathrm{h}}$ 1.0

Half credit lower limit	Full credit range	Half credit upper limit
45 h	46 h to 50 h	51 h

For CME-2:

$v_{1,\mathrm{m}} = 384.941\,\mathrm{km\,s^{-1}}$ which gives $\boxed{\tau_{2,\mathrm{m}} = 126\,\mathrm{h}}$ 1.0

Half credit lower limit	Full credit range	Half credit upper limit
116 h	119 h to 133 h	136 h

(D02.2b) The observed arrival times at Earth of CME-1 and CME-2 are 46.0 h and 74.5 h, respectively. The empirical model is considered to be VALID for a particular CME if its predicted arrival time is within 20% of its observed arrival time; otherwise, it is NOT VALID. Indicate the validity of the model for each CME by ticking (✓) the appropriate box in the Summary Answersheet. **2**

Solution:
The arrival time for CME-1 is $\tau_{1,\mathrm{m}} = 48.0\,\mathrm{h}$,
therefore the percentage error in the arrival time of CME-1 is,
$\Delta\tau_{1,\mathrm{m}} = \frac{48.0-46.0}{46.0} \times 100\% = 4.35\%$

Similarly, the arrival time for CME-2 is $\tau_{2,\mathrm{m}} = 126\,\mathrm{h}$,
hence the percentage error is arrival time for CME-2 is,

$\Delta\tau_{2,\mathrm{m}} = \frac{126-74.5}{74.5} \times 100\% = 69.1\%$

	VALID	NOT VALID
CME-1	✓	
CME-2		✓

2.0

CMEs in presence of solar wind

In reality, the space between the Sun and the Earth is permeated with the solar wind, which exerts a drag force on CMEs. This drag force can either decelerate or accelerate a CME, depending on the CME's velocity relative to that of the solar wind. To account for the solar wind's influence, we will use a "drag-only" model for distances $R_{\mathrm{obs}}(t) \geq R_0$, where R_0 is the distance beyond which the drag force becomes the dominant force affecting the CME's motion.

The distance from the surface of the Sun as determined from the "drag-only" model, $R_{\mathrm{D}}(t)$, and velocity, $V_{\mathrm{D}}(t)$, of a CME in this model is given by

$$R_{\mathrm{D}}(t) = \frac{S}{\gamma} \ln\left[1 + S\gamma(V_0 - V_{\mathrm{s}})(t - t_0)\right] + V_{\mathrm{s}}(t - t_0) + R_0$$

$$V_{\mathrm{D}}(t) = \frac{V_0 - V_{\mathrm{s}}}{1 + S\gamma(V_0 - V_{\mathrm{s}})(t - t_0)} + V_{\mathrm{s}}$$

where, $\gamma = 2 \times 10^{-8}\,\mathrm{km}^{-1}$, V_{s} is the constant speed of the solar wind, R_0 and V_0 are the distance and velocity, respectively, at time t_0, and S is the sign factor. $S = 1$ if $V_0 > V_{\mathrm{s}}$; $S = -1$ if $V_0 \leq V_{\mathrm{s}}$.

(D02.3) The tables below show the observed radial distance from the surface of the Sun, $R_{\mathrm{obs}}(t)$ (measured in $\mathrm{R}_\odot$), as a function of time, t (in hours), for two CMEs: CME-3 and CME-4. The last data point in each table (D5 and P8, respectively) corresponds to the arrival time of the respective CME at Earth. For this part, assume $V_{\mathrm{s}} = 330\,\mathrm{km\,s^{-1}}$.

CME-3		
Data point	t (in h)	$R_{\mathrm{obs}}(t)$ (in $\mathrm{R}_\odot$)
D1	0.200	6.36
D2	0.480	7.99
D3	1.22	11.99
D4	1.49	13.51
D5	58.05	214

CME-4		
Data point	t (in h)	$R_{\mathrm{obs}}(t)$ (in $\mathrm{R}_\odot$)
P1	1.00	4.00
P2	3.00	6.00
P3	4.00	9.00
P4	5.00	11.0
P5	21.0	43.0
P6	50.0	100
P7	85.0	170
P8	111	214

We shall evaluate if the "drag-only" model satisfactorily predicts the arrival times of these CMEs. To use this model an appropriate choice of t_0, and corresponding R_0 and V_0 needs to be made.

(D02.3a) For CME-3, take the following two cases: **6**

(C1) t_0 is taken as the midpoint of the interval D1 – D2

(C2) t_0 is taken as the midpoint of the interval D3 – D4

Assume the velocity remains constant in each specific interval D1–D2 and D3–D4, but may differ between the two intervals.

Using t_0, R_0, and V_0, calculate the difference between the observed and the predicted radial distance, $\delta R_{\mathrm{D}} \equiv R_{\mathrm{obs}}(t) - R_{\mathrm{D}}(t)$ in units of $\mathrm{R}_\odot$ at $t = 58.05\,\mathrm{h}$, for each of the two cases.

Solution:

Inserting, R_0, t_0, V_0, and V_s in the given expression

$$R_D(t) = \frac{S}{\gamma}\ln\left[1 + S\gamma(V_0 - V_s)(t - t_0)\right] + V_s(t - t_0) + R_0,$$

we get

Case C1:

$$R_0 = \frac{7.99 + 6.36}{2}$$

$$R_0 = 7.18\,\mathrm{R}_\odot$$ 0.5

$$t_0 = \frac{0.20 + 0.48}{2} = 0.340\,\mathrm{h}$$ 0.5

$$V_0 = \frac{(7.99 - 6.36) \times 695700}{(0.48 - 0.20) \times 3600}$$

$$V_0 = 1.12 \times 10^3\,\mathrm{km\,s^{-1}}$$ 1.0

we get, $R_D = 210.61\,\mathrm{R}_\odot$ at $t = 58.05$ h. 0.5

Thus, $\boxed{\delta R_D = 214 - 210.61 = 3.39\,\mathrm{R}_\odot}$. 0.5

Half credit lower limit	Full credit range	Half credit upper limit
	$3.30\,\mathrm{R}_\odot$ to $3.80\,\mathrm{R}_\odot$	

Case C2:

$$R_0 = \frac{11.99 + 13.51}{2} = 12.8\,\mathrm{R}_\odot$$ 0.5

$$t_0 = \frac{1.22 + 1.49}{2} = 1.36\,\mathrm{h}$$ 0.5

$$V_0 = \frac{(13.51 - 11.99) \times 695700}{(1.49 - 1.22) \times 3600}$$

$$V_0 = 1.09 \times 10^3\,\mathrm{km\,s^{-1}}$$ 1.0

Using the expression above, $R_D = 210.86\,\mathrm{R}_\odot$ at t=58.05. 0.5

Thus, $\boxed{\delta R_D = 214 - 210.86 = 3.14\,\mathrm{R}_\odot}$. 0.5

Half credit lower limit	Full credit range	Half credit upper limit
	$2.90\,\mathrm{R}_\odot$ to $3.20\,\mathrm{R}_\odot$	

(D02.3b) Evaluate $R_D(t)$ at points, P5, P6, P7, and P8 between the Sun and the Earth for CME-4 for the following two cases adopting the procedure similar to (D02.3a): **4**
(C3) t_0 is taken as the midpoint of the interval P1 – P2
(C4) t_0 is taken as the midpoint of the interval P3 – P4

Solution:

Case C3:

$$R_0 = \frac{6 + 4}{2} = 5\,\mathrm{R}_\odot$$

$$t_0 = \frac{3 + 1}{2} = 2\,\mathrm{h}$$

$$V_0 = \frac{(6 - 4) \times 695700}{(3 - 1) \times 3600} = 193\,\mathrm{km\,s^{-1}}$$ 1.0

Case C4:

$$R_0 = \frac{11+9}{2} = 10\,\mathrm{R}_\odot$$

$$t_0 = \frac{5+4}{2} = 4.5\,\mathrm{h}$$

$$V_0 = \frac{(11-9)\times 695700}{(5-4)\times 3600} = 386\,\mathrm{km\,s^{-1}}$$

1.0

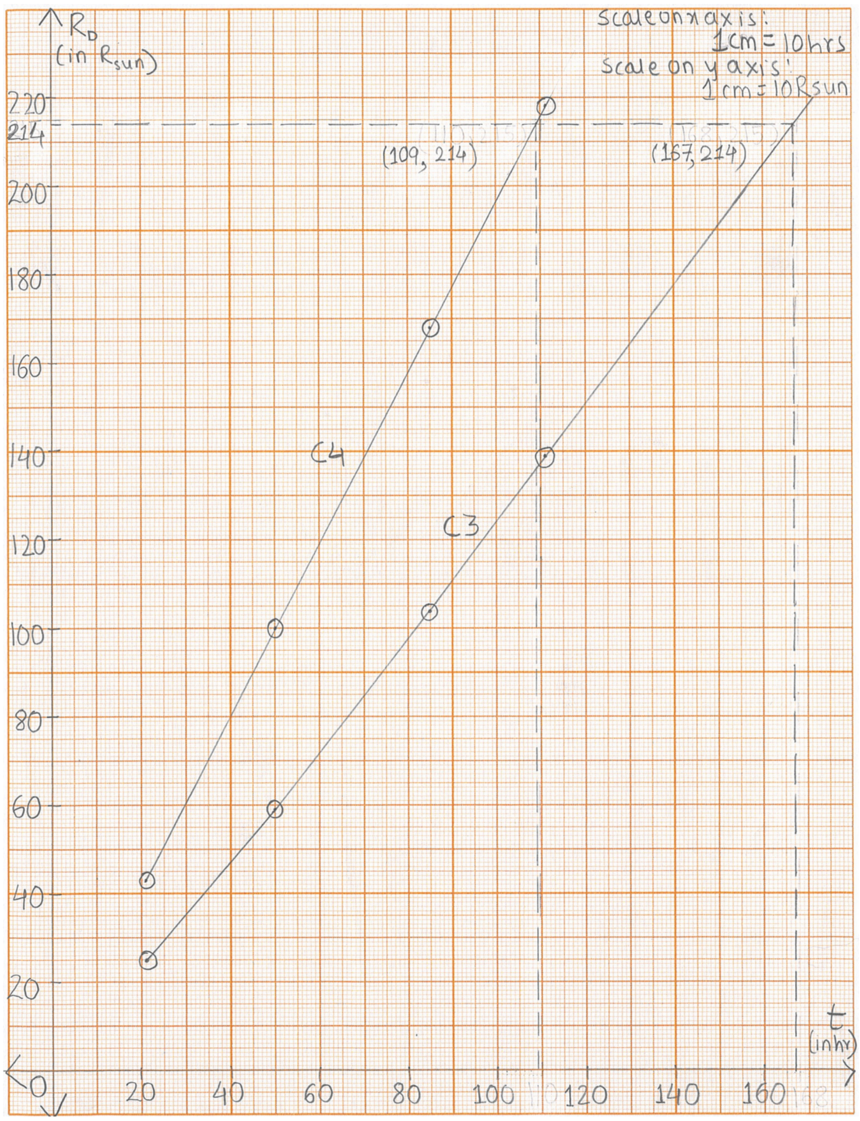
Data Point	t (in h)	C3	C4
		$R_\mathrm{D}(t)$ (in $\mathrm{R}_\odot$)	$R_\mathrm{D}(t)$ (in $\mathrm{R}_\odot$)
P5	21.0	25.1	42.8
P6	50.0	59.1	99.9
P7	85.0	104	168
P8	111	139	218

2.0

0.5 marks deducted for every second wrong entry

(D02.3c) Plot $R_\mathrm{D}(t)$ (in $\mathrm{R}_\odot$) vs t (in hours) for the two cases, C3 and C4, for CME-4 at points, P5, P6, P7, and P8 (mark your graph as "D02.3c"). On the same graph, draw smooth curves of $R_\mathrm{D}(t)$ for the above mentioned two cases. For this part, take the range of x axis from 0 to 180 h. **10**

Solution:
The plot below shows the data points of CME-4 and smooth variation of R_D extrapolated up to $220\,\mathrm{R}_\odot$.



Labels with units of both R_D and t axes		0.5
Either scale mentioned or ticks marked on both axes		1.0
More than 75% of the graph paper used		0.5
All 8 points marked correctly		4.0
6 to 7 points marked correctly	3.0	
4 to 5 points marked correctly	2.0	
$\leq$4 points marked correctly	0.0	
Smooth curve passing through all points for R_D (2 marks for each case)		4.0

If the curve is not smooth, a penalty of 1.0 mark for each case.
If the curve is not passing through the plotted points, a penalty of 1.0 mark for each case.

(D02.3d) Using the graph, estimate the absolute difference, $|\delta\tau|$ between the actual arrival time of CME-4 at the Earth and its time of arrival predicted by the drag-only model, for each of the cases C3 and C4. **4**

Solution:
The arrival time of CME at Earth ($214\,\mathrm{R}_\odot$) can be obtained by estimating the point of intersection between $R_\mathrm{D} = 214\,\mathrm{R}_\odot$ = constant and two smooth curves.

Correctly extrapolating the curve for case C3	1.0
Identifying the intersection of smooth curves and $R_\mathrm{D} = 214\,\mathrm{R}_\odot$	1.0

The points of intersection are 167 h and 109 h for the cases C3 and C4, respectively.
Thus, absolute time differences are:
$\boxed{\text{C3: } |\delta\tau| = 56\,\mathrm{h}}$ 1.0

Half credit lower limit	Full credit range	Half credit upper limit
	54 h to 58 h	

and
$\boxed{\text{C4: } |\delta\tau| = 2\,\mathrm{h}}$. 1.0

Half credit lower limit	Full credit range	Half credit upper limit
	0 h to 4 h	

(D02.3e) Indicate whether the following statement is TRUE or FALSE by ticking (✓) the appropriate box in the Summary Answersheet (no written justification needed): "The drag forces exerted by the solar wind on CMEs become dominant for CME-3 at an earlier time compared to CME-4." **1**

Solution:
The statement is $\boxed{\text{TRUE}}$. 1.0
In the scenario of CME-3, the "drag-only" model predicts the arrival distance accurately when R_D is computed using the initial velocity, V_0 and initial distance, R_0 at 0.20 h and beyond.
While for CME-4, the "drag-only" model start accurately predicting the arrival times of CME-4 when R_D is calculated using V_0 and R_0 at 4 h after the launch of the CME.

(D02.4) Consider drag as the dominant force acting on 10 CMEs in part D02.1. Assume that the "drag-only" model is applicable from the surface of the Sun ($R_0 = 1\,\mathrm{R}_\odot$) and beyond, for all CMEs. Estimate and tabulate the solar wind speed V_s in $\mathrm{km\,s^{-1}}$ for each CME. Further, estimate the average solar wind speed $V_\mathrm{s,\,avg}$ for all 10 CMEs. **7**

Solution:

$$V_\mathrm{D}(t) = \frac{V_0 - V_\mathrm{s}}{1 + S\gamma(V_0 - V_\mathrm{s})(t - t_0)} + V_\mathrm{s}$$

$$v = \frac{u - V_\mathrm{s}}{1 + S\gamma(u - V_\mathrm{s})\tau} + V_\mathrm{s}.$$ 1.0

Taking, $S = 1$ if $v < u$, else $S = -1$ The above expression is quadratic equation, thus, we get two values of V_s. If $v < u$, choose the solution with $V_\mathrm{s} < u$. If $v > u$, choose the solution with $V_\mathrm{s} > u$.
Calculating V_s using the above conditions for all ten CMEs.

5.0

CME Name	u ($\mathrm{km\,s^{-1}}$)	v ($\mathrm{km\,s^{-1}}$)	τ (h)	V_s ($\mathrm{km\,s^{-1}}$)
CME-A	804	470	74.5	337
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CME-C	523	396	103.5	314
CME-D	830	415	71.0	270
CME-E	665	400	104.5	303
CME-F	347	350	101.5	369
CME-G	446	375	99.5	305
CME-H	155	360	97.0	457
CME-I	1016	515	67.0	357
CME-J	683	410	54.0	248

Correctly calculating the V_s for each CME	0.5

Taking the average of the solar wind speed, V_s, for each case, we get

$\boxed{V_{s,\,avg} = 339\,\mathrm{km\,s^{-1}}}$ 1.0

Half credit lower limit	Full credit range	Half credit upper limit
	$338\,\mathrm{km\,s^{-1}}$ to $340\,\mathrm{km\,s^{-1}}$	