

Short Problems: (10 points each)

Problem 1: Calculate the mean mass density for a super massive black hole with total mass of $1 \times 10^8 M_{\odot}$ inside the Schwarzschild radius.

Problem 2: Estimate the number of photons per second that arrive on our eye at $\lambda = 550 \text{ nm}$ (V-band) from a G2 main sequence star with apparent magnitude of $m = 6$ (the threshold of naked eye visibility). Assume the eye pupil diameter is 6 mm and all the radiation from this star is in $\lambda = 550 \text{ nm}$.

Problem 3: Estimate the radius of a planet that a man can escape its gravitation by jumping vertically. Assume density of the planet and the Earth are the same.

Problem 4: In a typical Persian architecture, on top of south side windows there is a structure called "Tabeshband" (shader), which controls sunlight in summer and winter. In summer when the Sun is high, Tabeshband prevents sunlight to enter rooms and keeps inside cooler. In the modern architecture it is verified that the Tabeshband saves about 20% of energy cost. Figure (1) shows a vertical section of this design at latitude of $36^\circ.0 \text{ N}$ with window and Tabeshband.

Using the parameters given in the figure, calculate the maximum width of the Tabeshband, " x ", and maximum height of the window, " h " in such a way that:

- No direct sunlight can enter to the room in the summer solstice at noon.
- The direct sunlight reaches the end of the room (indicated by the point **A** in the figure) in the winter solstice at noon.

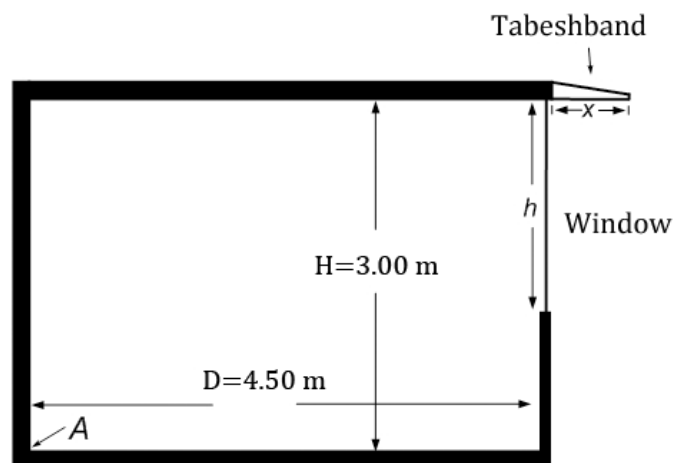


Figure (1)

Problem 5: The Damavand Mountain is located at the North part of Iran, in south coast of Caspian Sea. Consider an observer standing on the Damavand mountaintop (latitude = $35^{\circ}57'N$; longitude = $52^{\circ}6'E$; altitude $5.6 \times 10^3 \text{ m}$ from the mean sea level) and looking at the sky over the Caspian Sea. What is the minimum declination for a star, to be seen marginally circumpolar for this observer. Geodetic radius of the Earth at this latitude is 6370.8 km . Surface level of the Caspian Sea is approximately equal to the mean sea level.

Problem 6: Derive a relation for the escape velocity of an object, launched from the center of a proto-star cloud. The cloud has uniform density with the mass of M and radius R . Ignore collisions between the particles of the cloud and the launched object. If the object were allowed to fall freely from the surface, it would reach the center with a velocity equal to $\sqrt{\frac{GM}{R}}$.

Problem 7: A student tries to measure field of view (FOV) of the eyepiece of his/her telescope, using rotation of the Earth. To do this job, the observer points the telescope towards Vega (alpha Lyr., RA: 18.5^h , Dec: $+39^{\circ}$), turns off its "clock drive" and measures trace out time, $t=5.3$ minutes, that Vega crosses the full diameter of the FOV. What is the FOV of this telescope in arc-minutes?

Problem 8: Estimate the mass of a globular cluster with the radius of $r = 20 \text{ pc}$ and root mean square velocity of stars equal to $v_{rms} = 3 \text{ kms}^{-1}$.

Problem 9: The Galactic longitude of a star is $l = 15^{\circ}$. Its radial velocity with respect to the Sun is $V_r = 100 \text{ kms}^{-1}$. Assume stars in the disk of the Galaxy are orbiting the center with a constant velocity of $V_0 = 250 \text{ kms}^{-1}$ in circular orbits in the same sense in the galactic plane. Calculate distance of the star from the center of the Galaxy.

Problem 10: A main sequence star with the radius and mass of $R = 4R_{\odot}$, $M = 6M_{\odot}$ has an average magnetic field of $1 \times 10^{-4} \text{ T}$. Calculate the strength of the magnetic field of the star when it evolves to a neutron star with the radius of 20 km .

Problem 11: Assume the mass of neutrinos is $m_{\nu} = 10^{-5}m_e$. Calculate the number density of neutrinos (n_{ν}) needed to compensate the dark matter of the universe. Assume the universe is flat and 25 % of its mass is dark matter.

Hint: Take the classical total energy equal to zero

Problem 12: Calculate how much the radius of the Earth's orbit increases as a result of the Sun losing mass due to the thermo-nuclear reactions in its center in 100 years. Assume the Earth's orbit remains circular during this period.

Problem 13: Assume that you are living in the time of Copernicus and do not know anything about Kepler's laws. You might calculate Mars-Sun distance in the same way as he did. After accepting the revolutionary belief that all the planets are orbiting around the Sun, not around the Earth, you measure that the orbital period of Mars is 687 days, then you observe that 106 days after opposition of Mars, the planet appears in quadrature. Calculate Mars-Sun distance in astronomical unit (AU).

Problem 14: A satellite is orbiting around the Earth in a circular orbit in the plane of the equator. An observer in Tehran at the latitude of $\varphi = 35.6^\circ$ observes that the satellite has a zenith angle of $z = 46.0^\circ$, when it transits the local meridian. Calculate the distance of the satellite from the center of the Earth (in the Earth radius unit).

Problem 15: An eclipsing close binary system consists of two giant stars with the same sizes. As a result of mutual gravitational force, stars are deformed from perfect sphere to the prolate spheroid with $a = 2b$, where a and b are semi-major and semi-minor axes (the major axes are always co-linear). The inclination of the orbital plane to the plane of sky is 90° . Calculate the amplitude of light variation in magnitude (Δm) as a result of the orbital motion of two stars. Ignore temperature variation due to tidal deformation and limb darkening on the surface of the stars.

Hint: A prolate spheroid is a geometrical shape made by rotating of an ellipse around its major axis, like rugby ball or melon.

Problem 16: High Altitude Projectile (45 points)

A projectile which initially is put on the surface of the Earth at the sea level is launched with the initial speed of $v_0 = \sqrt{GM/R}$ and with the projecting angle (with respect to the local horizon) of $\theta = \frac{\pi}{6}$. M and R are the mass and radius of the Earth respectively. Ignore the air resistance and rotation of the Earth.

- Show that the orbit of the projectile is an ellipse with a semi-major axis of $a = R$.
- Calculate the highest altitude of the projectile with respect to the Earth surface (in the unit of the Earth radius).
- What is the range of the projectile (distance between launching point and falling point)?
- What is eccentricity (e) of the ellipse?
- Find the flying time for the projectile.

Problem 17: Apparent number density of stars in the Galaxy (45 points)

Let us model the number density of stars in the disk of Milky Way Galaxy with a simple exponential function of $n = n_0 \exp(-\frac{r-R_0}{R_d})$, where r represents the distance from the center of the Galaxy, R_0 is the distance of the Sun from the center of the Galaxy, R_d is the typical size of disk and n_0 is the stellar density of disk at the position of the Sun. An astronomer observes the center of the Galaxy within a small field of view. We take a particular type of Red giant stars (red clump) as the standard candles for the observation with approximately constant absolute magnitude of $M = -0.2$,

- Considering a limiting magnitude of $m = 18$ for a telescope, calculate the maximum distance that telescope can detect the red clump stars. For simplicity we ignore the presence of interstellar medium so there is no extinction.
- Assume an extinction of 0.70 mag/kpc for the interstellar medium. Repeat the calculation as done in the part (a) and obtain a rough number for the maximum distance these red giant stars can be observed.
- Give an expression for the number of these red giant stars per magnitude within a solid angle of Ω that we can observe with apparent magnitude in the range of m and $m + \Delta m$, (i.e. $\frac{\Delta N}{\Delta m}$). Red giant stars contribute f of overall stars. In this part assume no extinction in the interstellar medium as part (a).
Hint : the Tylor expansion of $y = \log_{10} x$ is :

$$y = y_0 + \frac{1}{\ln 10} \frac{x - x_0}{x}$$